

The In-betweens

Not-whole numbers and their various forms

Booklet 1 (of 5)

In this booklet:

- Different things fractions can represent
- Part of a group – multiplying an integer by a fraction
- Equivalent (equal) fractions
- Simplifying fractions
- Improper fractions and mixed numbers

Student Name: _____

Teacher Name: _____

Class: _____

Commencement date: _____



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Not-whole numbers and their forms

Booklet 1 (of 5)

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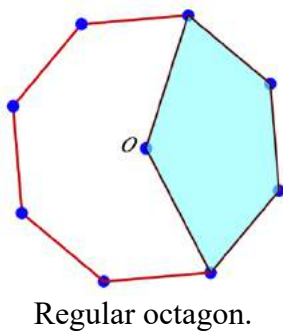
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1. Fractions as part(s) of a whole – part 1

Question 1

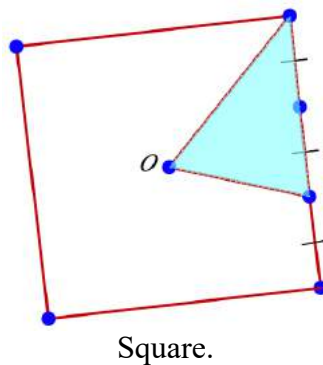
State what fraction of the whole area of each shape is shaded.

a)



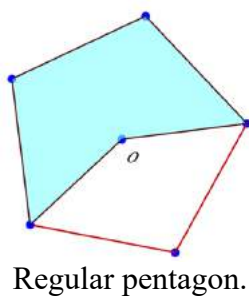
Fraction shaded =

d)



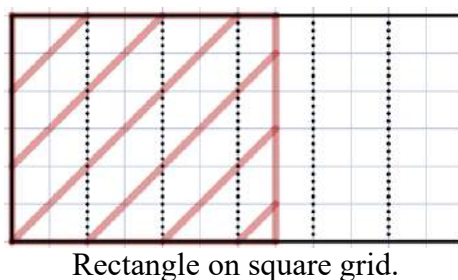
Fraction shaded =

b)



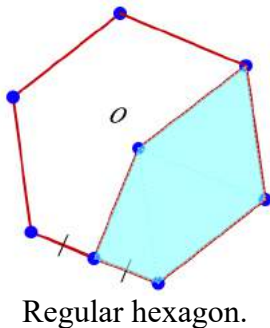
Fraction shaded =

e)



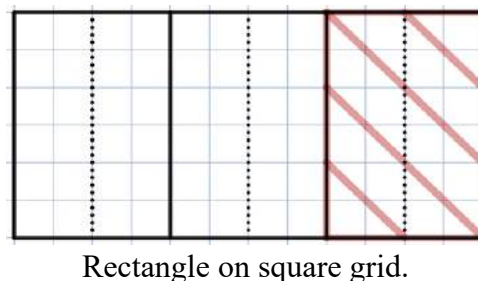
Fraction shaded =

c)



Fraction shaded =

f)

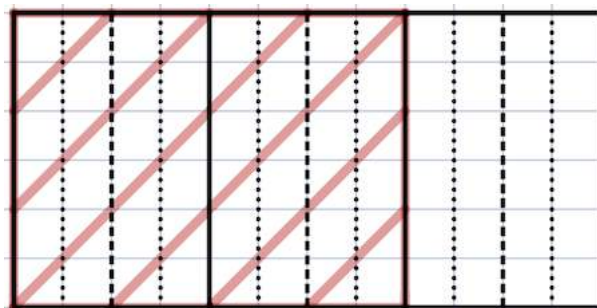


Fraction shaded =

Question 2

State what fraction of the whole area of each shape is shaded.
(There is more than one correct answer. 😊)

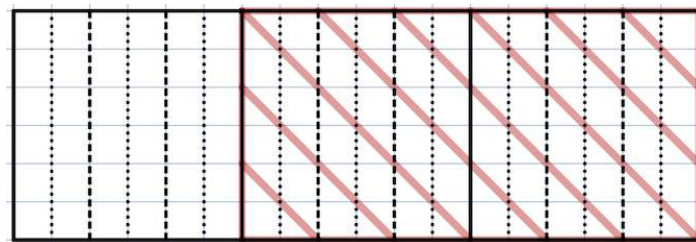
a)



Rectangle on square grid.

Fraction shaded =

b)



Rectangle on square grid.

Fraction shaded =

c)



Rectangle on square grid.

Fraction shaded =

Question 3

Write in the missing numerators that make each statement true.

a) $\frac{2}{3} = \frac{\quad}{6} = \frac{\quad}{12}$

b) $\frac{2}{3} = \frac{\quad}{9} = \frac{\quad}{18}$

c) $\frac{3}{4} = \frac{\quad}{12} = \frac{\quad}{36}$

d) $\frac{3}{4} = \frac{\quad}{20} = \frac{\quad}{100}$

e) $\frac{5}{8} = \frac{\quad}{24} = \frac{\quad}{32}$

f) $\frac{3}{7} = \frac{\quad}{42} = \frac{\quad}{63}$

Question 4

Write in the missing denominators that make each statement true.

a) $\frac{2}{5} = \frac{4}{\quad} = \frac{8}{\quad}$

b) $\frac{1}{4} = \frac{7}{\quad} = \frac{21}{\quad}$

c) $\frac{3}{5} = \frac{15}{\quad} = \frac{30}{\quad}$

d) $\frac{5}{6} = \frac{30}{\quad} = \frac{35}{\quad}$

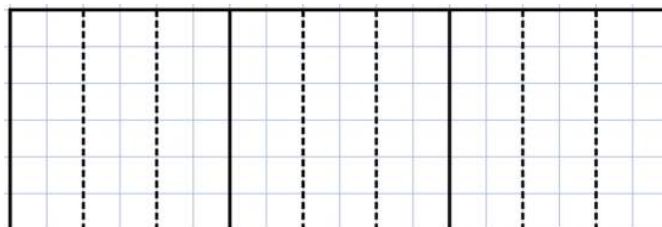
e) $\frac{3}{8} = \frac{21}{\quad} = \frac{27}{\quad}$

f) $\frac{6}{7} = \frac{24}{\quad} = \frac{54}{\quad}$

2. Fractions as part(s) of a whole – part 2

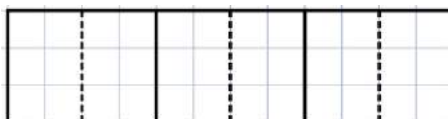
Question 1

- a) Shade $\frac{12}{18}$ of the whole rectangle (on square grid).



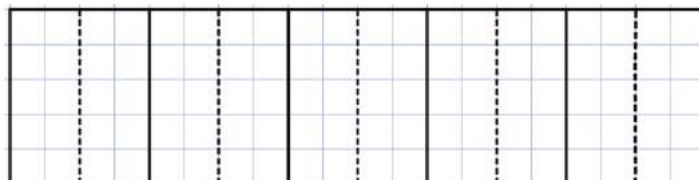
Notice anything?

- b) Shade $\frac{4}{12}$ of the whole rectangle (on square grid).



Notice anything?

- c) Shade $\frac{8}{20}$ of the whole rectangle (on square grid).



Notice anything?

- d) Shade $\frac{5}{20}$ of the whole rectangle (on square grid).



Notice anything?

- e) Shade $\frac{12}{21}$ of the whole rectangle (on square grid).



Notice anything?

Question 2

Write in the missing numerators that make each statement true.

a) $\frac{12}{18} = \frac{\quad}{9} = \frac{\quad}{3}$

b) $\frac{4}{12} = \frac{\quad}{6} = \frac{\quad}{3}$

c) $\frac{8}{20} = \frac{\quad}{10} = \frac{\quad}{5}$

d) $\frac{5}{20} = \frac{\quad}{4}$

e) $\frac{12}{21} = \frac{\quad}{7}$

f) $\frac{18}{42} = \frac{\quad}{21} = \frac{\quad}{7}$

Question 3

Write in the missing denominators that make each statement true.

a) $\frac{14}{42} = \frac{2}{\quad} = \frac{1}{\quad}$

b) $\frac{15}{60} = \frac{5}{\quad} = \frac{1}{\quad}$

c) $\frac{8}{12} = \frac{4}{\quad} = \frac{2}{\quad}$

d) $\frac{12}{16} = \frac{3}{\quad}$

e) $\frac{30}{36} = \frac{5}{\quad}$

f) $\frac{60}{75} = \frac{12}{\quad} = \frac{4}{\quad}$

3. Fractions as part(s) of a whole – part 3

Question 1

The region between two circles that share the same centre (concentric circles) is called an *annulus*.

You can see an annulus, coloured red, in Figure 1.

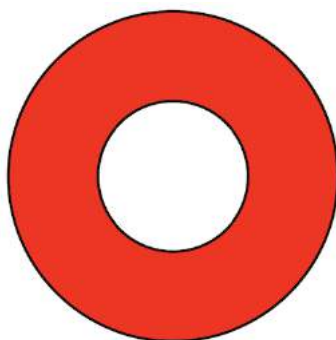
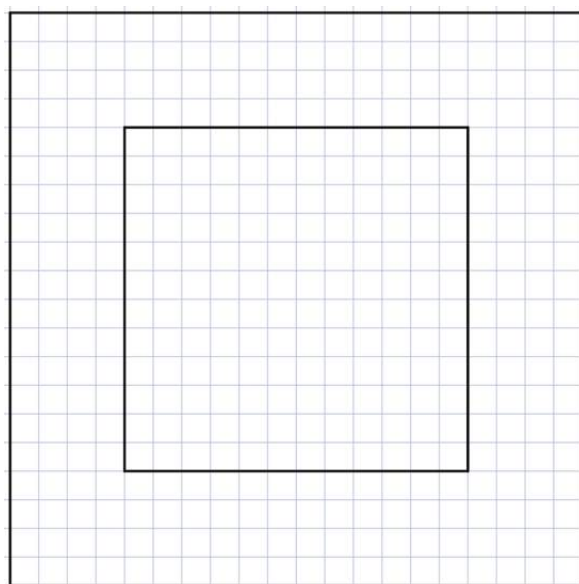


Figure 1.

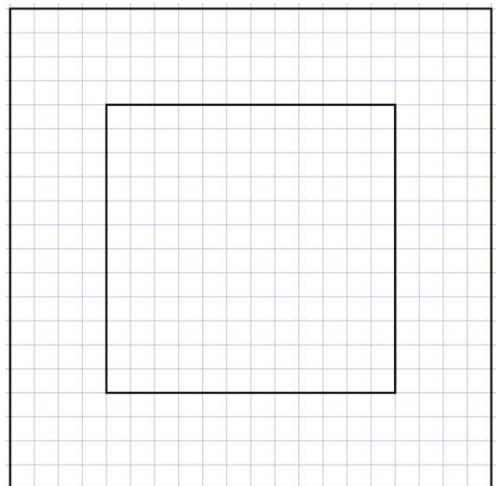
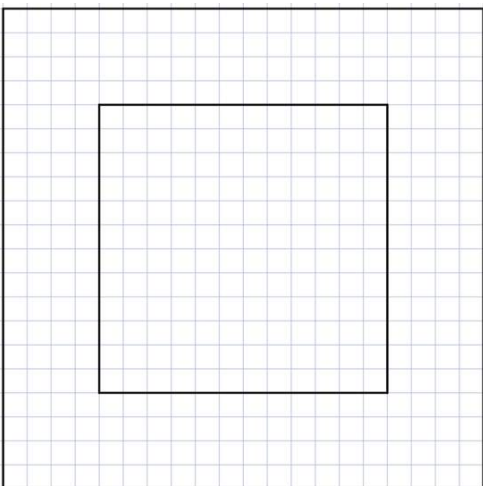
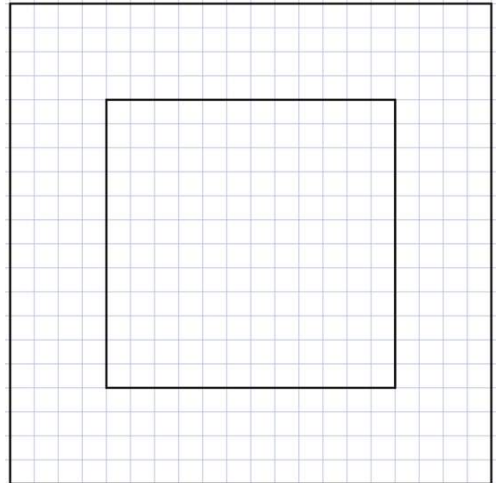
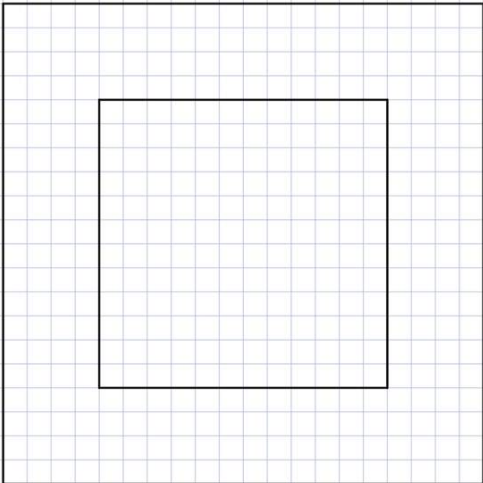
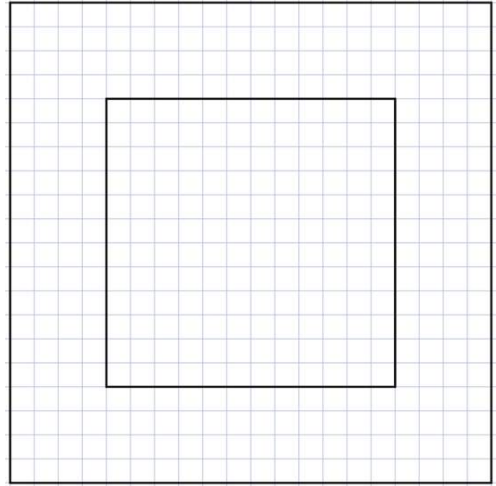
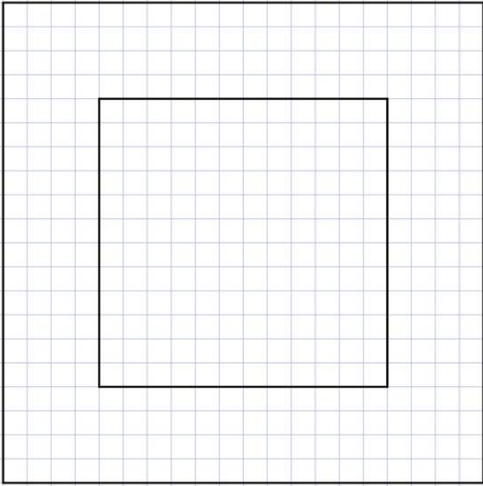
Below are two concentric squares.

The region between two concentric squares does not seem to have a name, so henceforth such a region shall be named *squannulus*.

Your job is to colour exactly $\frac{1}{4}$ of the *squannulus*, in the most creative and colourful way you can.



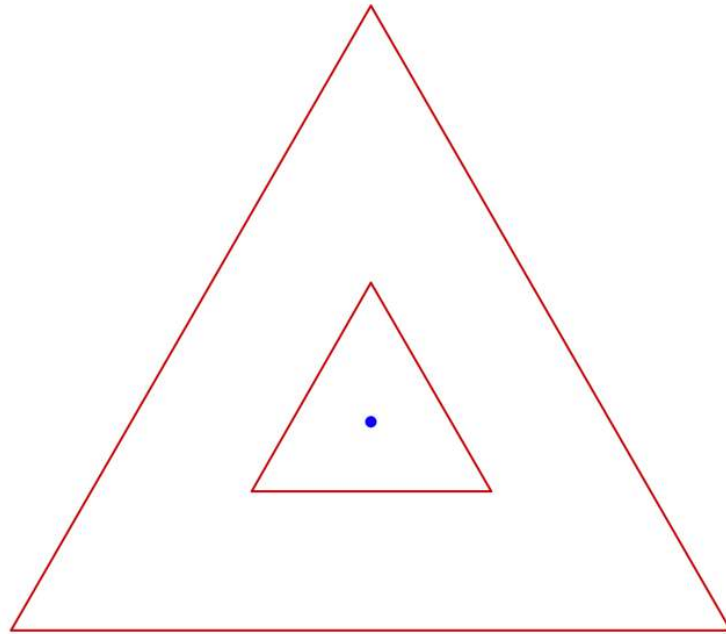
Some more *squannuli* are provided on the next page so you can cut loose and colour exactly $\frac{1}{4}$ of the *squannulus* in a variety of different, colourful and creative ways.



Question 2

Below are two concentric *equilateral* triangles.

- The small equilateral triangle has side length of 1 unit.
- The large equilateral triangle has side length of 3 units.
- The corresponding sides of the small and large triangle are parallel.
- The triangles' common centre is marked with a blue point.



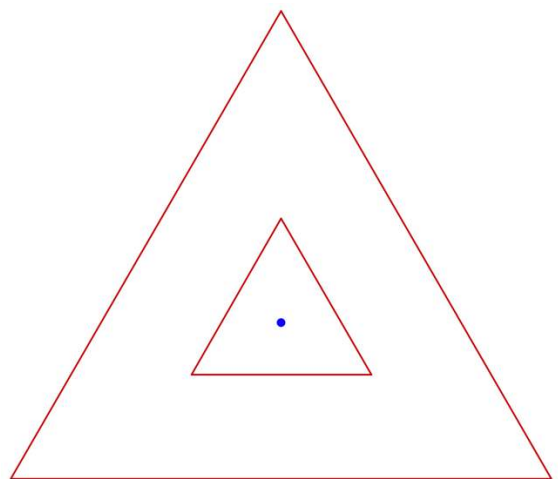
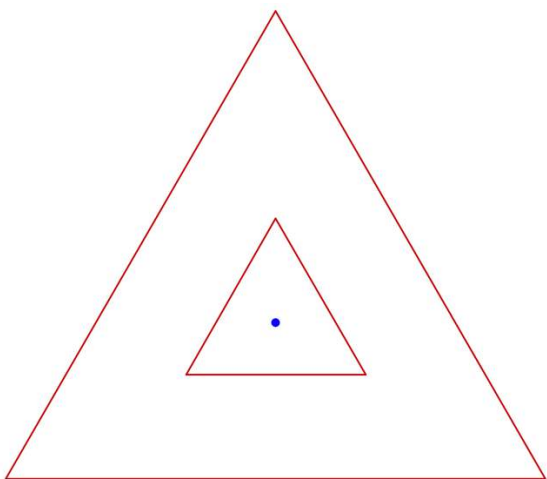
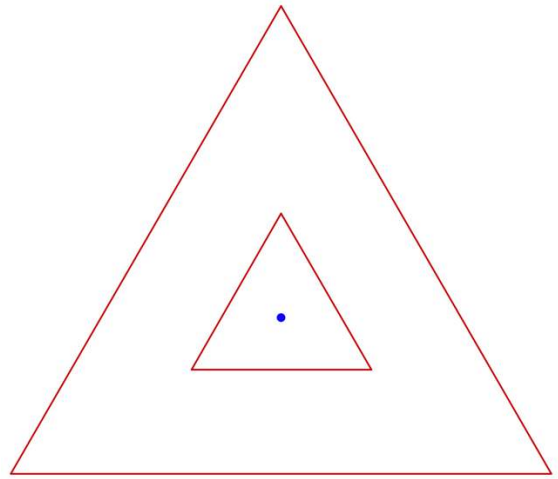
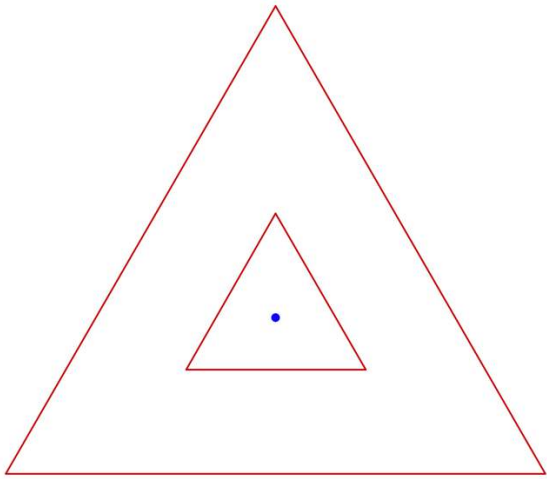
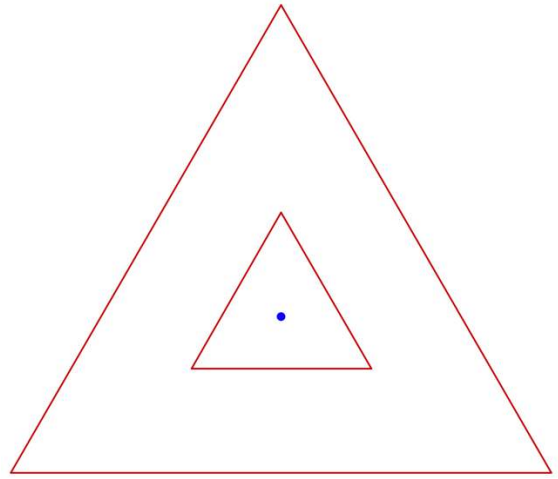
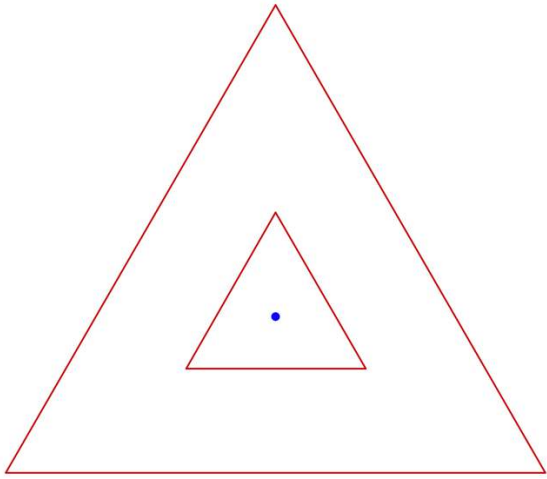
The region between two concentric equilateral triangles does not seem to have a name, so henceforth such a region shall be named *triangulus*.

Your first job is to colour exactly $\frac{1}{4}$ of a *triangulus*, in the most creative and colourful way you can.

Some more *trianguli* are provided on the next page so you can cut loose and colour exactly $\frac{1}{4}$ of the *triangulus* in a variety of different, colourful, and creative ways.

Your second job is to colour exactly $\frac{3}{4}$ of a *triangulus* in a symmetrical manner.

Make it lovely and colourful. The best one wins a prize! 😊



4. Fractions as part(s) of a whole – part 4

Question 1

Figure 2 shows a representation of a tangram puzzle on a square grid. A tangram is a square cut into various shapes.

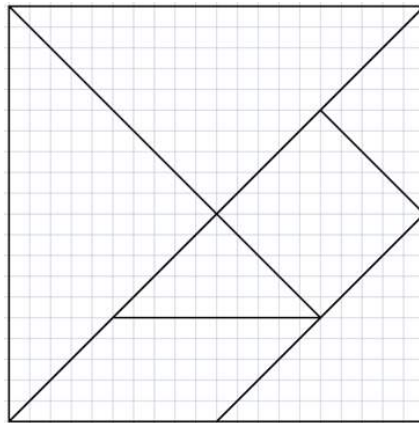
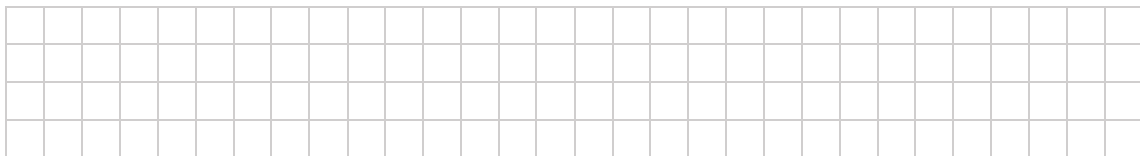


Figure 2. (Copies of this figure can be found on page 17.)

What fraction of the area of the square is occupied by the two smallest right triangles?



Question 2

Figure 3 shows two identical regular hexagons enclosed in a rectangle.

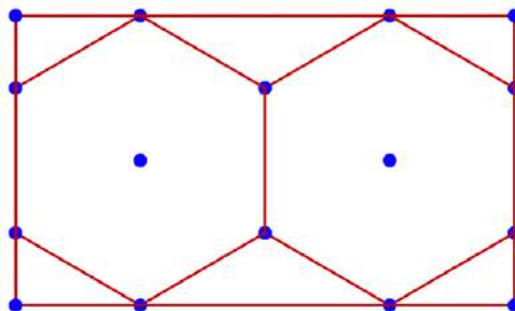
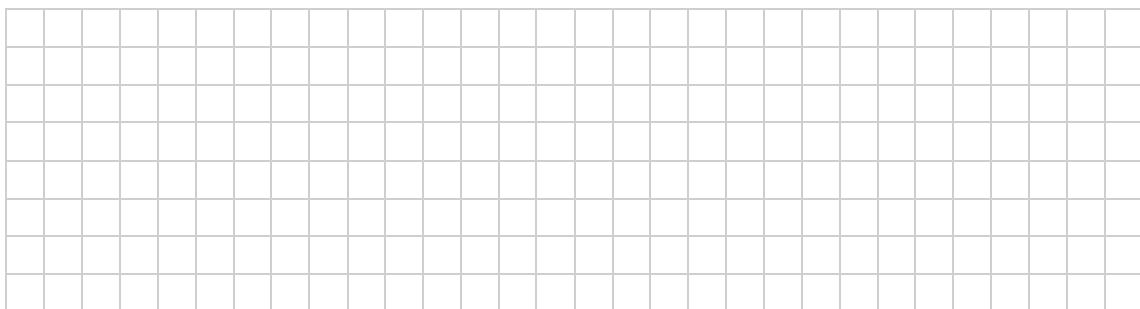


Figure 3. (Copies of this figure can be found on page 17.)

What fraction of the rectangle's area is occupied by the two regular hexagons?



Question 3

Figure 4 shows two identical regular hexagons enclosed in a rectangle.

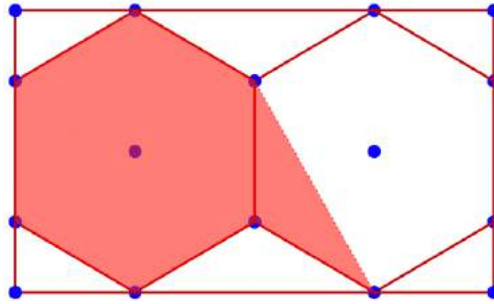
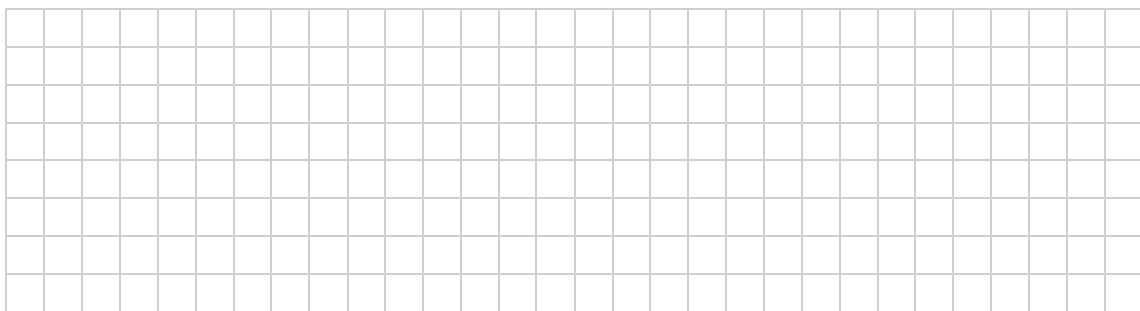


Figure 4. (Copies of this figure can be found on page 17.)

What fraction of the rectangle's area is shaded?



Question 4

Figure 5 shows a square with each side divided, identically, into four equal segments.

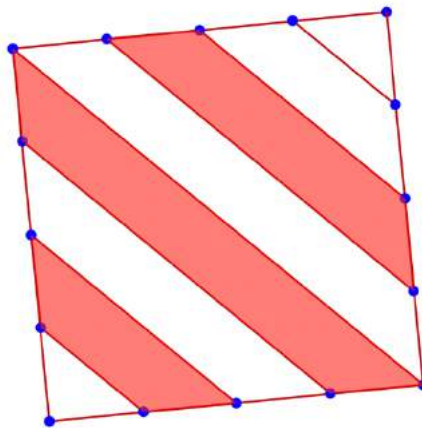
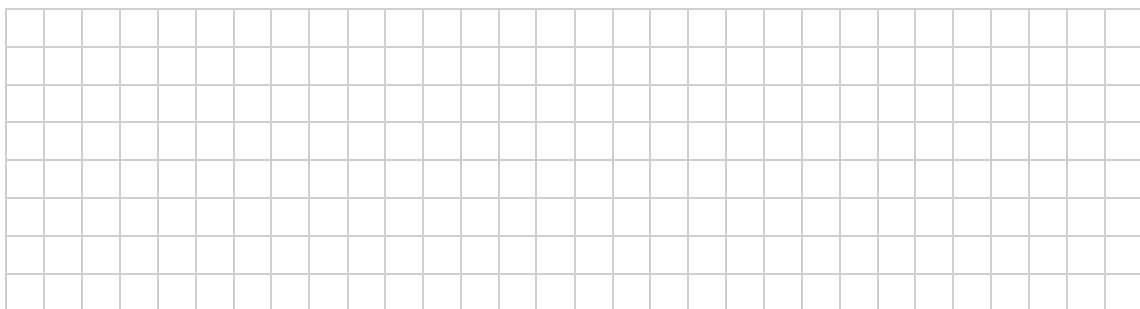


Figure 5. (Copies of this figure can be found on page 17.)

What fraction of the square's area is shaded?



Question 5

Figure 6 shows four small identical equilateral triangles packed within one large equilateral triangle. The centres of each triangle are marked.

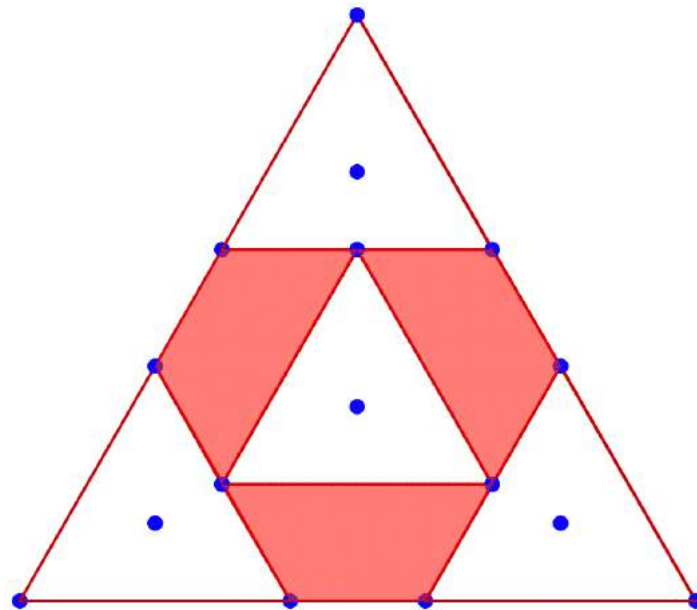


Figure 6. (Copies of this figure can be found on page 17.)

What fraction of the large equilateral triangle is shaded?



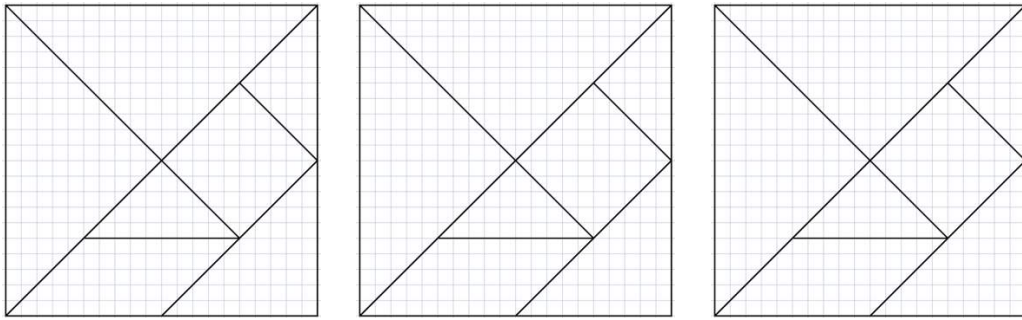


Figure 2.

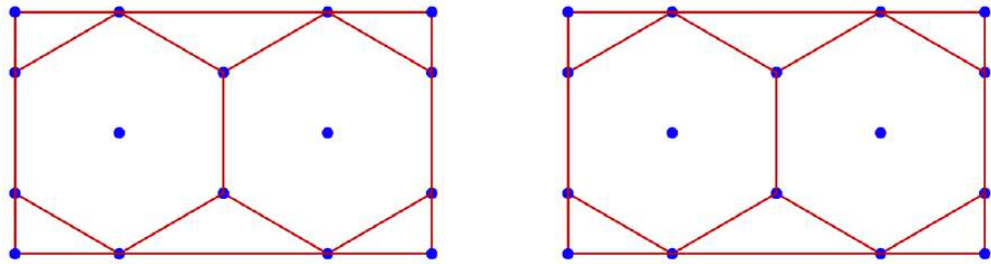


Figure 3.

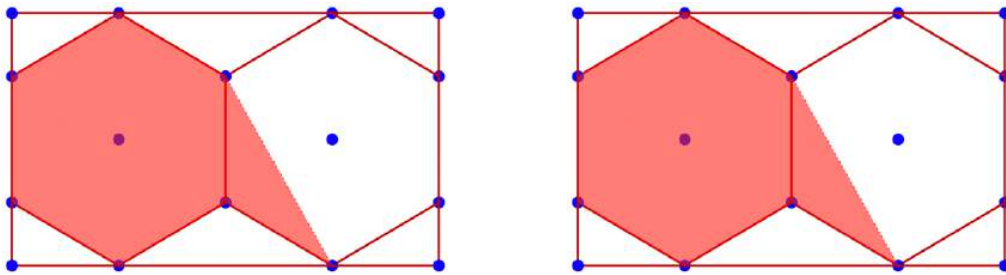


Figure 4.

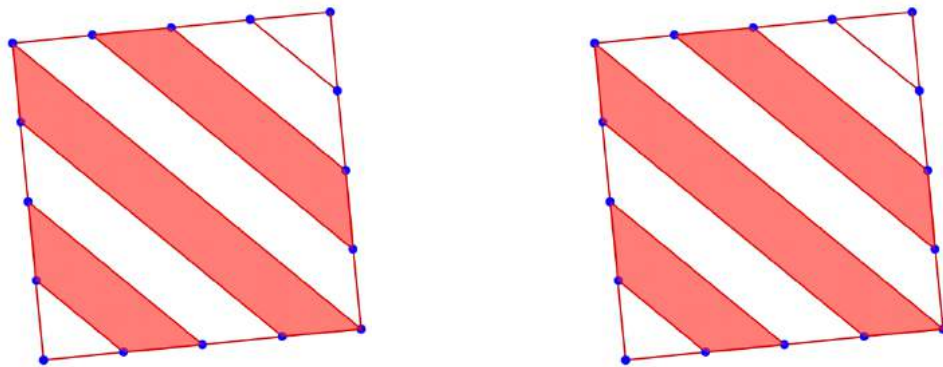


Figure 5.

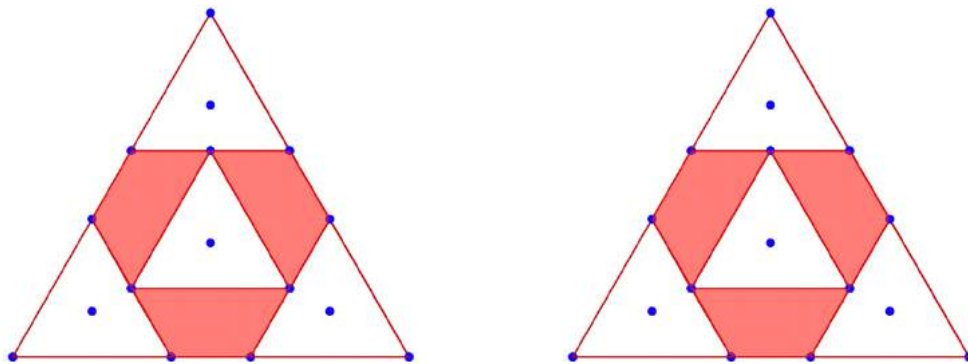
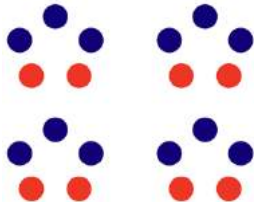
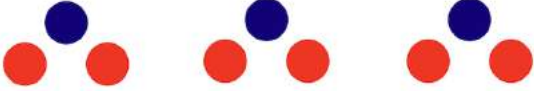
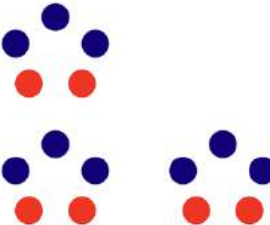
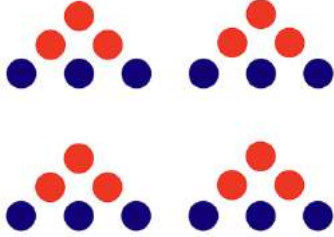
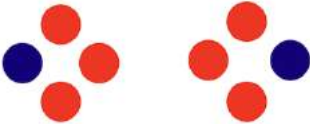


Figure 6.

5. Fractions as “for every”

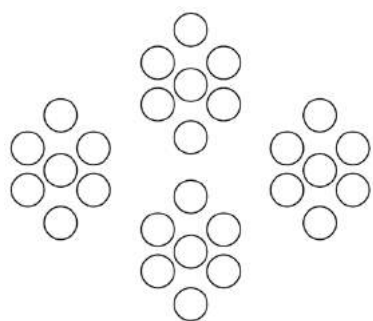
Question 1

State what *fraction* of the *total number of dots* are *blue*.

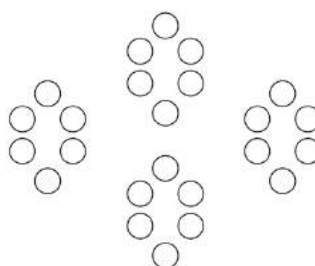
a)  Fraction of all dots that are blue =	e)  Fraction of all dots that are blue =
b)  Fraction of all dots that are blue =	f)  Fraction of all dots that are blue =
c)  Fraction of all dots that are blue =	g)  Fraction of all dots that are blue =
d)  Fraction of all dots that are blue =	h)  Fraction of all dots that are blue =

Question 2

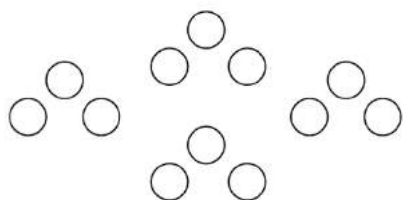
a) Colour $\frac{3}{7}$ of all these dots.



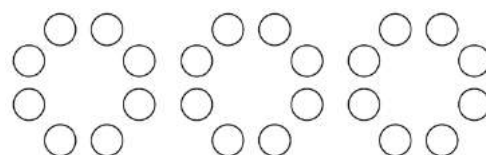
e) Colour $\frac{2}{3}$ of all these dots.



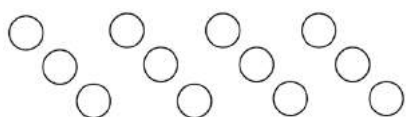
b) Colour $\frac{2}{3}$ of all these dots.



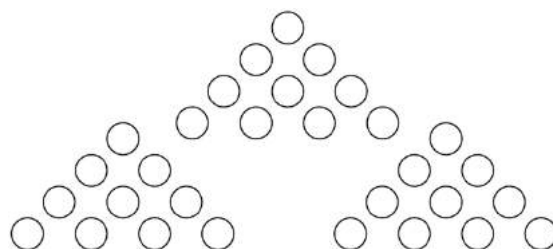
f) Colour $\frac{3}{4}$ of all these dots.



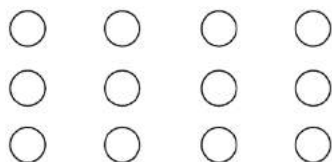
c) Colour $\frac{2}{3}$ of all these dots.



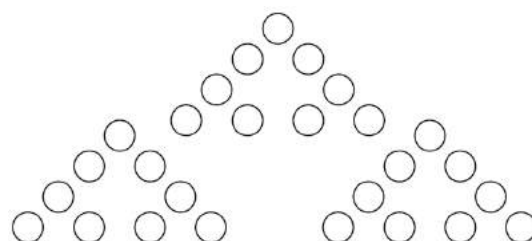
g) Colour $\frac{3}{5}$ of all these dots.



d) Colour $\frac{2}{3}$ of all these dots.



h) Colour $\frac{2}{3}$ of all these dots.



Question 3

Write in the missing numerators that make each statement true.

a) $\frac{3}{7} = \frac{\quad}{21} = \frac{\quad}{28}$

b) $\frac{2}{3} = \frac{\quad}{12} = \frac{\quad}{24}$

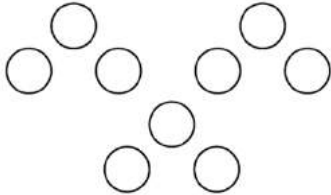
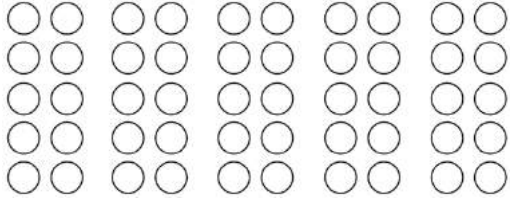
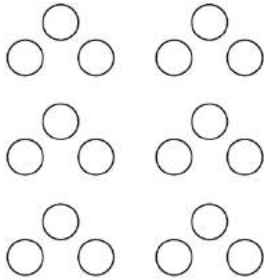
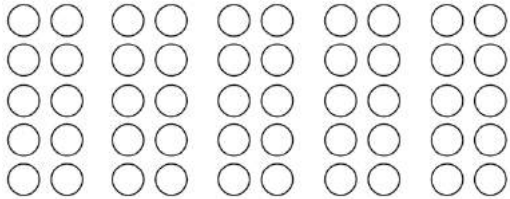
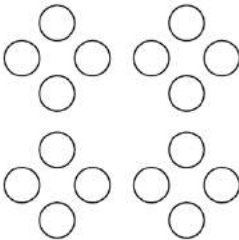
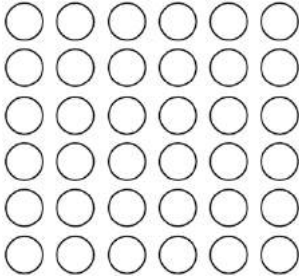
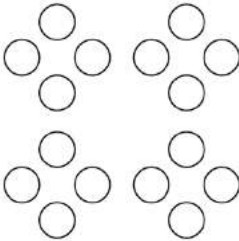
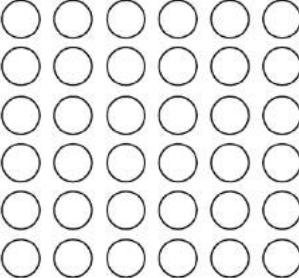
c) $\frac{3}{4} = \frac{\quad}{24} = \frac{\quad}{32}$

d) $\frac{3}{5} = \frac{\quad}{20} = \frac{\quad}{30}$

e) $\frac{4}{9} = \frac{\quad}{27} = \frac{\quad}{45}$

f) $\frac{5}{6} = \frac{\quad}{18} = \frac{\quad}{24}$

Question 4

a) Colour $\frac{6}{9}$ of all these dots. 	e) Colour $\frac{8}{10}$ of all these dots. 
b) Colour $\frac{6}{9}$ of all these dots. 	f) Colour $\frac{8}{10}$ of all these dots, in a different way than you did in part e). 
c) Colour $\frac{6}{8}$ of all these dots. 	g) Colour $\frac{6}{18}$ of all these dots. 
d) Colour $\frac{6}{8}$ of all these dots, in a different way than you did in part c). 	h) Colour $\frac{18}{72}$ of all these dots. 

Question 5

Write in the missing numerators that make each statement true.

a) $\frac{6}{9} = \frac{\quad}{3}$

b) $\frac{6}{8} = \frac{\quad}{4}$

c) $\frac{8}{10} = \frac{\quad}{5}$

d) $\frac{6}{18} = \frac{\quad}{3}$

e) $\frac{18}{72} = \frac{\quad}{36} = \frac{\quad}{4}$

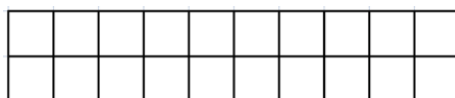
6. Part of a group - multiplication

6.1 Way of thinking #1

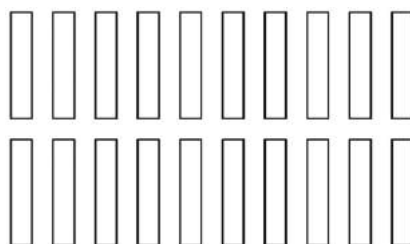
$2 \times 10 = 20$. We can think about 2×10 as,

2 groups of 10 objects.

We can represent it visually as an array, something like this,



or



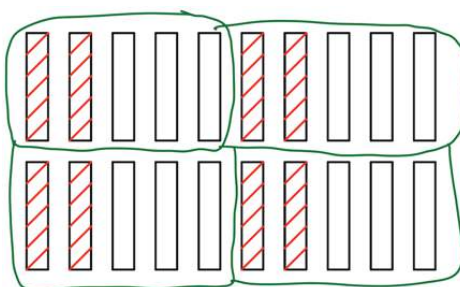
Instead of wanting 2 groups of 10 suppose we wanted,

$\frac{2}{5}$ of one group of 20 objects.

One way to think about this is that

we need 2 for every 5 of the 20 objects,

which can look like this,



So, we can write,

$$\frac{2}{5} \times 20 = 8$$

One way to *calculate* $\frac{2}{5} \times 20$ (without a picture) is to:

- first figure out how many groups of 5 are in 20,
 $20 \div 5 = 4$
- then determine how many objects in total are in 4 lots of 2,
 $4 \times 2 = 8$.

Example 1

Calculate $\frac{2}{3} \times 12$.

$$\begin{aligned}\frac{2}{3} \times 12 \\&= 2 \times 4 \quad (\text{since } 12 \div 3 = 4) \\&= 8\end{aligned}$$

Example 2

Calculate $\frac{5}{7} \times 42$.

$$\begin{aligned}\frac{5}{7} \times 42 \\&= 5 \times 6 \quad (\text{since } 42 \div 7 = 6) \\&= 30\end{aligned}$$

Question 1

Calculate each of the following.

a) $\frac{3}{4} \times 12$

e) $\frac{2}{3} \times 27$

b) $\frac{2}{7} \times 42$

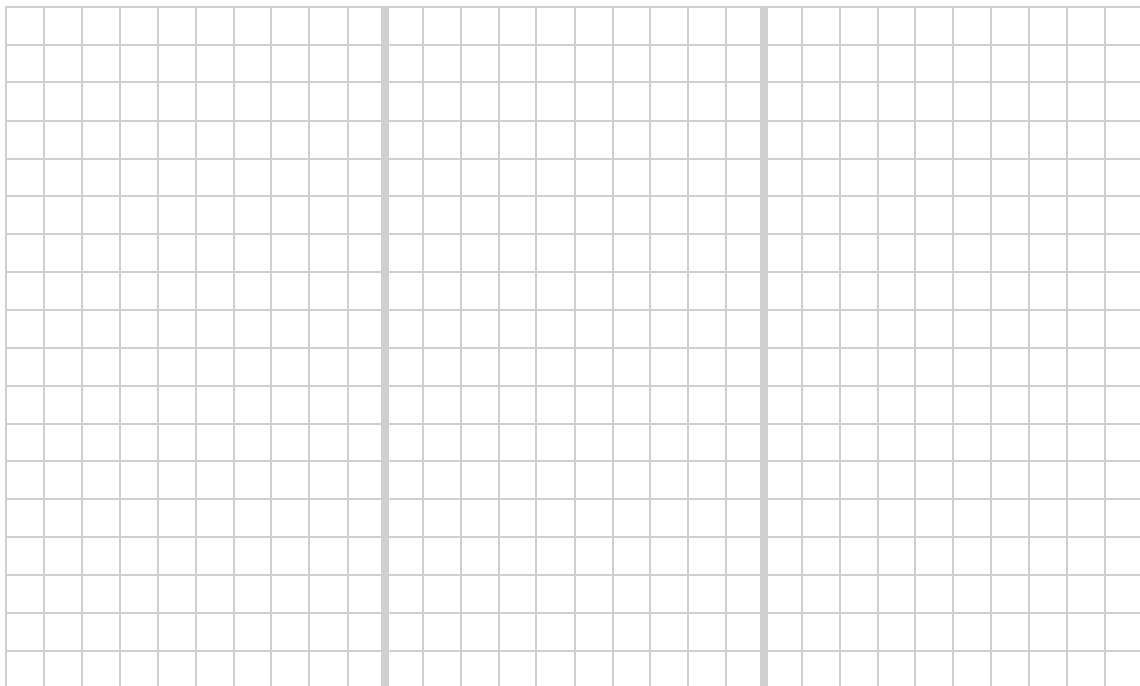
f) $\frac{5}{8} \times 56$

c) $\frac{2}{5} \times 30$

g) $\frac{7}{11} \times 77$

d) $\frac{3}{5} \times 45$

h) $\frac{7}{8} \times 72$

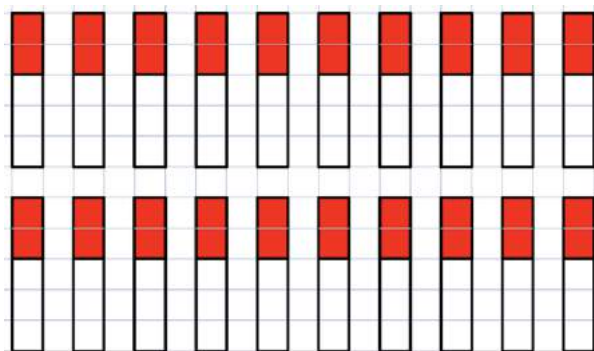


6.2 Way of thinking #2

A different, but equal, way to think about $\frac{2}{5} \times 20$ is as,

$\frac{2}{5}$ of *every one* of the 20 objects.

Or in other words 20 lots of $\frac{2}{5}$ (of a whole/object), which could be pictured like this,



In total we have *forty, one-fifth* pieces, or $\frac{40}{5}$; can you see them?

Or, if we put the pieces together in wholes, we have 8 whole objects.

Convince yourself that all the red pieces fit exactly into 8 whole objects.

So, to calculate $\frac{2}{5} \times 20$ we could do the following,

$$\begin{aligned}\frac{2}{5} \times 20 \\&= \frac{2 \times 20}{5} \\&= \frac{40}{5} \\&= 8\end{aligned}$$

To calculate $\frac{5}{7} \times 42$,

$$\begin{aligned}\frac{5}{7} \times 42 \\&= \frac{5 \times 42}{7} \\&= \frac{210}{7} \\&= 30\end{aligned}$$

Question 1

Calculate each of the following, using the method shown on the previous page.

a) $\frac{2}{3} \times 15$

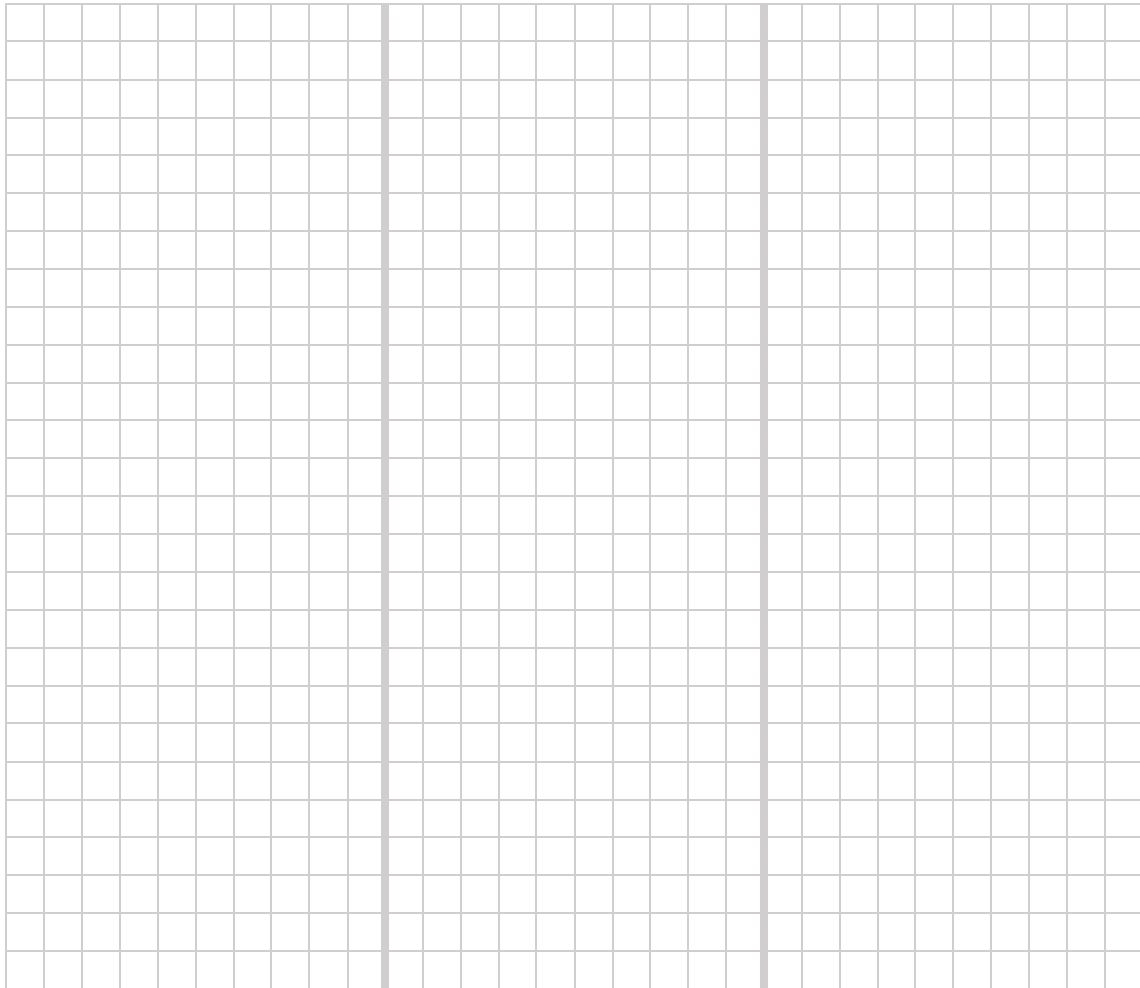
d) $\frac{5}{6} \times 12$

b) $\frac{3}{5} \times 40$

e) $\frac{5}{6} \times 24$

c) $\frac{2}{3} \times 45$

f) $\frac{2}{7} \times 14$



6.3 When there are *left-overs*

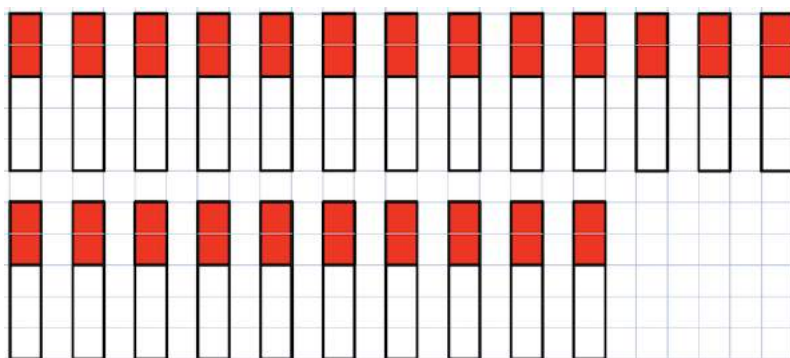
Suppose we need to calculate $\frac{2}{5}$ of one group of 23 objects.

We do not have a whole number of *groups of 5*. ☹

Mmm, the “2 for every 5” way of thinking is a bit tough. (But it can be done.)

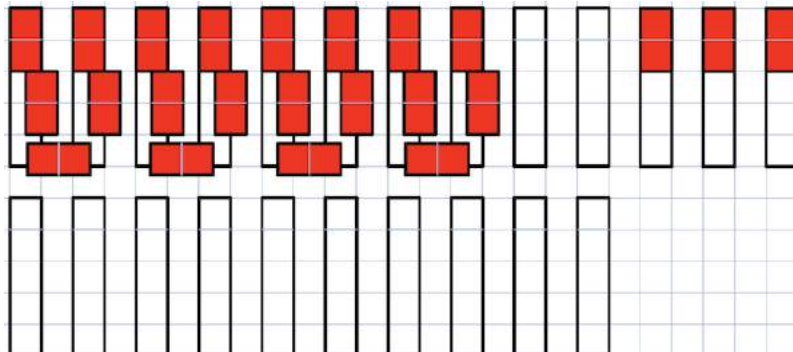
So, let’s think about it as $\frac{2}{5}$ of *every one* of the 23 objects.

Which could look like this:



How many whole objects?

The next diagram shows the start of moving the red two-fifth sized pieces around.



Convince yourself that there are

9 whole red objects plus $\frac{1}{5}$ of a whole red object.

$$\text{So, } \frac{2}{5} \times 23 = 9 + \frac{1}{5}$$

$9 + \frac{1}{5}$ is usually shortened to $9\frac{1}{5}$ (which is called a *mixed number*).

So, to calculate $\frac{2}{5} \times 23$ we could do the following,

$$\begin{aligned}\frac{2}{5} \times 23 \\ = \frac{46}{5} \text{ (forty-six fifths)}\end{aligned}$$

Now calculate how many 5s are in 46 (to fill up the wholes),

$$46 \div 5 = 9 \text{ remainder } 1.$$

The 1 is, in fact, *one-fifth* of a whole of 5. Therefore, in this case, 9 remainder 1 = $9\frac{1}{5}$.

So,

$$\begin{aligned}\frac{2}{5} \times 23 \\ = \frac{46}{5} \\ = 9 \text{ remainder } 1 \\ = 9\frac{1}{5}\end{aligned}$$

Example 1

Calculate $\frac{2}{3} \times 17$.

$$\begin{aligned}\frac{2}{3} \times 17 \\ = \frac{34}{3} \\ = 11 \text{ remainder } 1 \\ = 11\frac{1}{3}\end{aligned}$$

Example 2

Calculate $\frac{3}{5} \times 16$.

$$\begin{aligned}\frac{3}{5} \times 16 \\ = \frac{48}{5} \\ = 9 \text{ remainder } 3 \\ = 9\frac{3}{5}\end{aligned}$$

Question 1

Calculate each of the following, using the method on the previous page.

a) $\frac{2}{3} \times 16$

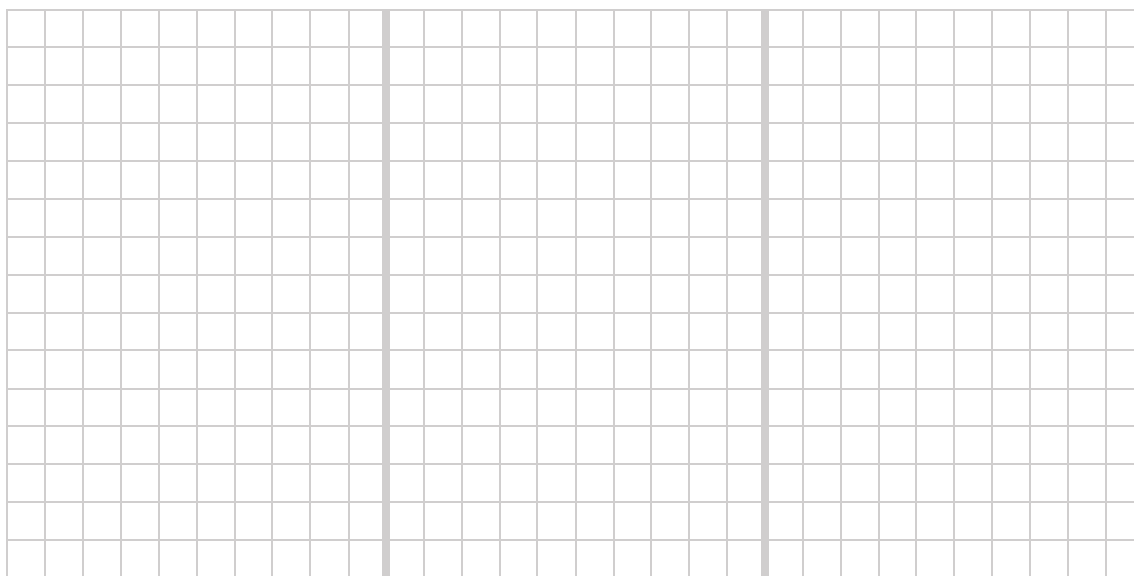
d) $\frac{5}{6} \times 11$

b) $\frac{3}{4} \times 10$

e) $\frac{2}{7} \times 17$

c) $\frac{2}{3} \times 19$

f) $\frac{3}{5} \times 42$



Question 2

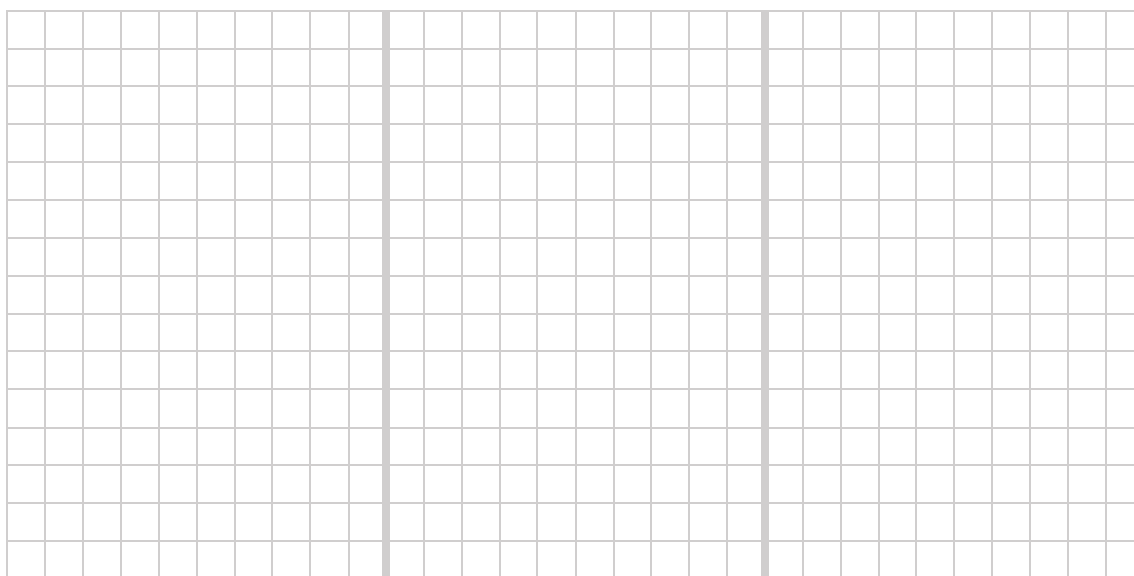
Calculate each of the following, using the method on the previous page.

a) $\frac{4}{7} \times 15$

c) $\frac{2}{3} \times 28$

b) $\frac{3}{8} \times 21$

d) $\frac{5}{6} \times 25$



7. Equivalent (equal) fractions – scaling up

Suppose a delicious block of chocolate is portioned so we can break it into *three* pieces of equal size.

If we ate *one* piece on Saturday and *one* piece on Sunday, then over the weekend,

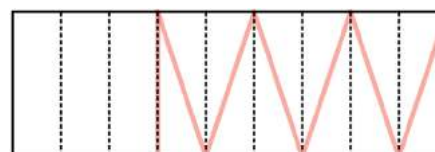
we ate $\frac{2}{3}$ of the block.



Instead, suppose *the same* block was portioned so we can break it into 9 pieces of equal size.

If we ate *three* on Saturday and *three* on Sunday, then over the weekend,

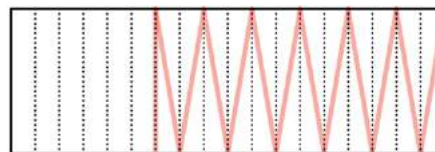
we ate $\frac{6}{9}$ of the block.



Instead, suppose *the same* block ... into 18 pieces of equal size.

If we ate *six* on Saturday and *six* on Sunday, then over the weekend,

we ate $\frac{12}{18}$ of the block.



Instead, suppose *the same* block ... into 108 pieces of equal size.

If we ate *thirty-six* on Saturday and *thirty-six* on Sunday, then over the weekend,

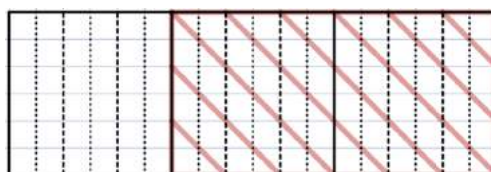
we ate $\frac{72}{108}$ of the block.



In each case **we ate the same amount of chocolate** and so:

$$\frac{2}{3} = \frac{6}{9} = \frac{12}{18} = \frac{72}{108} \text{ (of the block)}$$

We can picture the amount of chocolate eaten like this:



Multiplying the numerator and denominator of a given fraction *by the same number* will create a new fraction that looks different but that is *equal* to the given fraction.

Fractions that are equal, but have different numerators and denominators, are called *equivalent (equal) fractions*.

For example:

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} = \frac{9 \times 5}{12 \times 5} = \frac{45}{60} = \frac{45 \times 9}{60 \times 9} = \frac{405}{540} = \dots$$

$\frac{3}{4}, \frac{9}{12}, \frac{45}{60}, \dots$ all represent the same amount of a given whole and ultimately the same number on a number line.

We can create equivalent (equal) fractions by “scaling up” both the numerator and denominator of any fraction by the same factor.

Question 1

Draw a diagram that illustrates that:

a) $\frac{2}{5} = \frac{6}{15} = \frac{24}{60}$

b) $\frac{3}{4} = \frac{12}{16} = \frac{24}{32}$

c) $\frac{5}{6} = \frac{25}{30} = \frac{75}{90}$



Question 2

Make two *different* versions of each of the following statements.

A given fraction can only appear once in a set of four.

(Do not include any shown earlier in this section.)

a) $\frac{2}{3} = - = \frac{14}{-} = -$

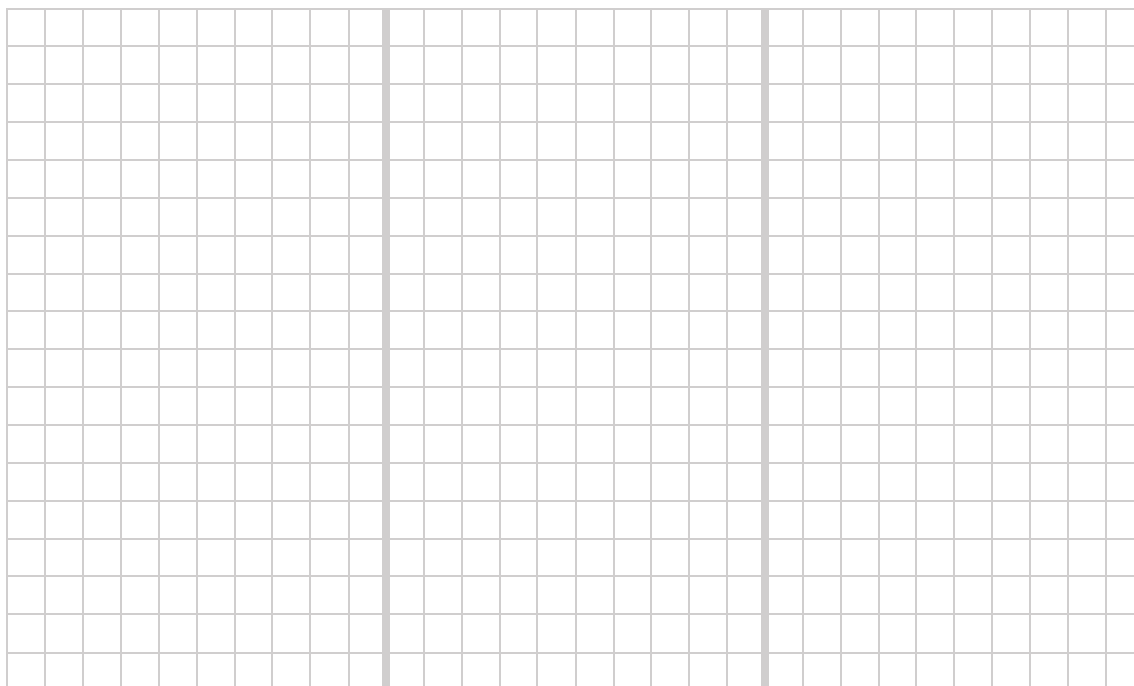
d) $\frac{2}{9} = - = - = \frac{-}{630}$

b) $\frac{3}{4} = - = \frac{-}{32} = -$

e) $\frac{3}{8} = - = \frac{27}{-} = -$

c) $\frac{3}{5} = - = \frac{-}{45} = -$

f) $\frac{7}{9} = - = \frac{63}{-} = -$



Question 3

Each black square is hiding a digit.

In each statement, no digit is repeated.

Find as many solutions as you can for each statement.

a) $\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare\blacksquare}$

d) $\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare\blacksquare} = \frac{\blacksquare\blacksquare}{\blacksquare\blacksquare}$

b) $\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare\blacksquare}{\blacksquare\blacksquare}$

e) $\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare\blacksquare} = \frac{\blacksquare\blacksquare}{\blacksquare\blacksquare\blacksquare}$

c) $\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare\blacksquare} = \frac{\blacksquare}{\blacksquare\blacksquare}$



8. Equivalent (equal) fractions – scaling down

We have learned that equivalent (equal) fractions can be created by “scaling-up” the numerator and denominator of a fraction by the same factor.

News flash!

We can also “scale-down” the numerator and denominator of a fraction, by the same factor, to create an equivalent (equal) fraction.

To do this we *divide* the numerator and denominator by the same number.

For example,

$$\frac{9}{15} = \frac{9 \div 3}{15 \div 3} = \frac{3}{5}$$

Note that both 3 and 5 are prime numbers and so we will not be able to find any more whole numbers that will divide into both the numerator and denominator of $\frac{3}{5}$.

Here is another example,

$$\frac{24}{44} = \frac{24 \div 4}{44 \div 4} = \frac{6}{11}$$

In this case, 11 is prime but 6 is not.

But there is no integer, other than 1, that will divide evenly into 6 and 11.

We say the *greatest common divisor (GCD)** of 6 and 11 is 1.

* You might know the GCD by another name, *the highest common factor* or HCF.

If a pair (or trio or ...) of integers has a *greatest common divisor* of 1 (**GCD = 1**), we say the numbers are *co-prime*.

Having a **GCD of 1** is the same as saying the numbers *do not share any common factors*.

Here is one last example,

$$\frac{12}{70} = \frac{12 \div 2}{70 \div 2} = \frac{6}{35}$$

There is no integer except 1 that divides into 6 and 35, so we say that

$\frac{6}{35}$ is in its *simplest form*.

A fraction is in *simplest form* when, the *largest* whole number that will divide into both the numerator and the denominator, with zero remainder, is 1.

Using different words to say the same thing, a fraction is in simplest form when, the *GCD of the numerator and the denominator is 1*.

To reduce a fraction to its *simplest form* we can find a number that will divide into both the numerator and the denominator, using our knowledge of the times tables.

To find the simplest form of $\frac{9}{24}$ we note that 3 divides into both 9 and 24, so:

$$\frac{9}{24} = \frac{9 \div 3}{24 \div 3} = \frac{3}{8}$$

Check that the GCD of 3 and 8 is 1, and it is, so $\frac{3}{8}$ is in simplest form.

To find the simplest form of $\frac{18}{24}$ we might first notice that both 18 and 24 are even.

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

It is not, as 3 will divide into both. So,

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

Check to see if the GCD of 3 and 4 is 1, and it is, so $\frac{3}{4}$ is in simplest form.

Question 1

a) $\frac{6}{8}$

b) $\frac{8}{12}$

c) $\frac{12}{42}$

d) $\frac{20}{25}$

Question 2

Write each of the following fractions in their simplest form.

a) $\frac{18}{30}$

c) $\frac{14}{35}$

e) $\frac{15}{45}$

g) $\frac{45}{30}$

b) $\frac{28}{63}$

d) $\frac{12}{42}$

f) $\frac{28}{56}$

h) $\frac{80}{16}$

8.2 Fractions to simplest form in *one-step*

Another way to reduce a fraction to its simplest form, is to find the greatest common divisor (GCD) of the numerator and the denominator and then

divide both the numerator and denominator by the GCD.

We can use our knowledge of the times tables to determine the GCD.

For example, $\text{GCD}(8 \text{ and } 12) = 4$ because $4 \times 2 = 8$, and $4 \times 3 = 12$, and 2 and 3 do not share any factors.

You may have memorised some common ones.

For more challenging cases, we can use the *prime factorisation* of the numerator and the denominator to determine the GCD.

Example 1.

Find the simplest form of $\frac{9}{24}$:

$$9 = 3 \times 3 \text{ and}$$

$$24 = 2 \times 2 \times 2 \times 3 \text{ and so}$$

$$\text{the GCD}(9, 24) = 3$$

$$\frac{9}{24} = \frac{9 \div 3}{24 \div 3} = \frac{3}{8}$$

Example 2.

Find the simplest form of $\frac{42}{90}$:

$$42 = 2 \times 3 \times 7 \text{ and}$$

$$90 = 2 \times 3 \times 3 \times 5 \text{ and so}$$

$$\text{the GCD}(42, 90) = 2 \times 3 = 6$$

$$\frac{42}{90} = \frac{42 \div 6}{90 \div 6} = \frac{7}{15}$$

Example 3.

Find the simplest form of $\frac{90}{126}$:

$$90 = 2 \times 3 \times 3 \times 5 \text{ and}$$

$$126 = 2 \times 3 \times 3 \times 7 \text{ and so}$$

$$\text{the GCD}(90, 126) = 2 \times 3 \times 3 = 18$$

$$\frac{90}{126} = \frac{90 \div 18}{126 \div 18} = \frac{90 \div 2 \div 3 \div 3}{126 \div 2 \div 3 \div 3} = \frac{5}{7}$$

or

$$\frac{90}{126} = \frac{90 \div 18}{126 \div 18} = \frac{90 \div 9 \div 2}{126 \div 2 \div 9} = \frac{5}{7}$$

Question 1

Calculate the GCD of

- c) 60 and 90

[illegible]

Question 2

Write each of the following fractions in their simplest form.

- g) $\frac{80}{32}$

- h) $\frac{105}{28}$

A full-page view of a blank sheet of graph paper. The page is covered by a uniform grid of thin, light gray horizontal and vertical lines, creating a series of small squares across the entire surface. There are no margins, text, or other markings on the paper.

Question 3

Calculate the GCD of

- a) 54 and 180

- b) 84 and 252

- c) 231 and 315

[illegible]

Question 4

Write each of the following fractions in their simplest form.

- a) $\frac{54}{180}$

- c) $\frac{231}{315}$

- e) $\frac{351}{56}$

- b) $\frac{84}{252}$

- d) $\frac{231}{252}$

- f) $\frac{12345}{678}$

9. Improper fractions to mixed numbers (fractions as division)

Suppose Kim said,

“Please give me *eight-halves* of a certain type of chocolate bar.”.

If the amount of chocolate was all that mattered, you might, quite rightly, give Kim 4 whole bars.

Eight-halves can be expressed as the fraction $\frac{8}{2}$ and so:

$$\frac{8}{2} = 4$$

Note that $8 \div 2 = 4$.

The fraction $\frac{a}{b}$ is equal to the division, $a \div b$.

So, twelve-fifths, or $\frac{12}{5}$, is the same as $12 \div 5$.

So, we can reason as follows:

$$\begin{aligned}\frac{12}{5} \\ &= 12 \div 5 \\ &= 2 \text{ remainder } 2^* \\ &= 2\frac{2}{5}\end{aligned}$$

* The remainder of 2, is $\frac{2}{5}$ of a whole of 5.

Fractions with a *numerator larger in value than the denominator* are called **improper fractions**.

Improper fractions can be converted to *mixed numbers* using division, as shown above.

Example 1.

Convert $\frac{24}{7}$ to a mixed number.

$$\begin{aligned}\frac{24}{7} \\ &= 24 \div 7 \\ &= 3 \text{ remainder } 3 \\ &= 3\frac{3}{7}\end{aligned}$$

Question 1

Convert each improper fraction to a mixed number.

a) $\frac{17}{2}$

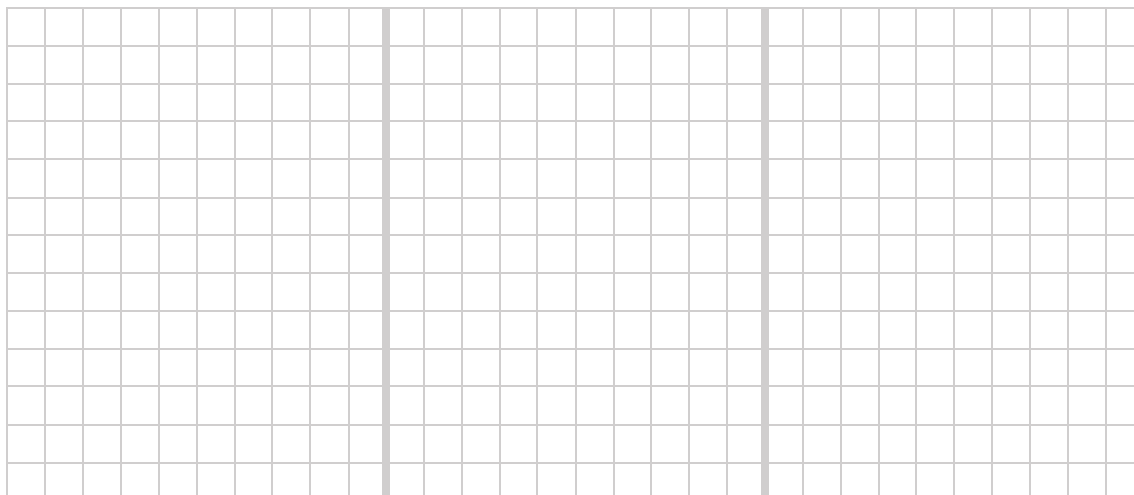
c) $\frac{34}{4}$

e) $\frac{61}{9}$

b) $\frac{28}{3}$

d) $\frac{59}{7}$

f) $\frac{93}{12}$



Question 2

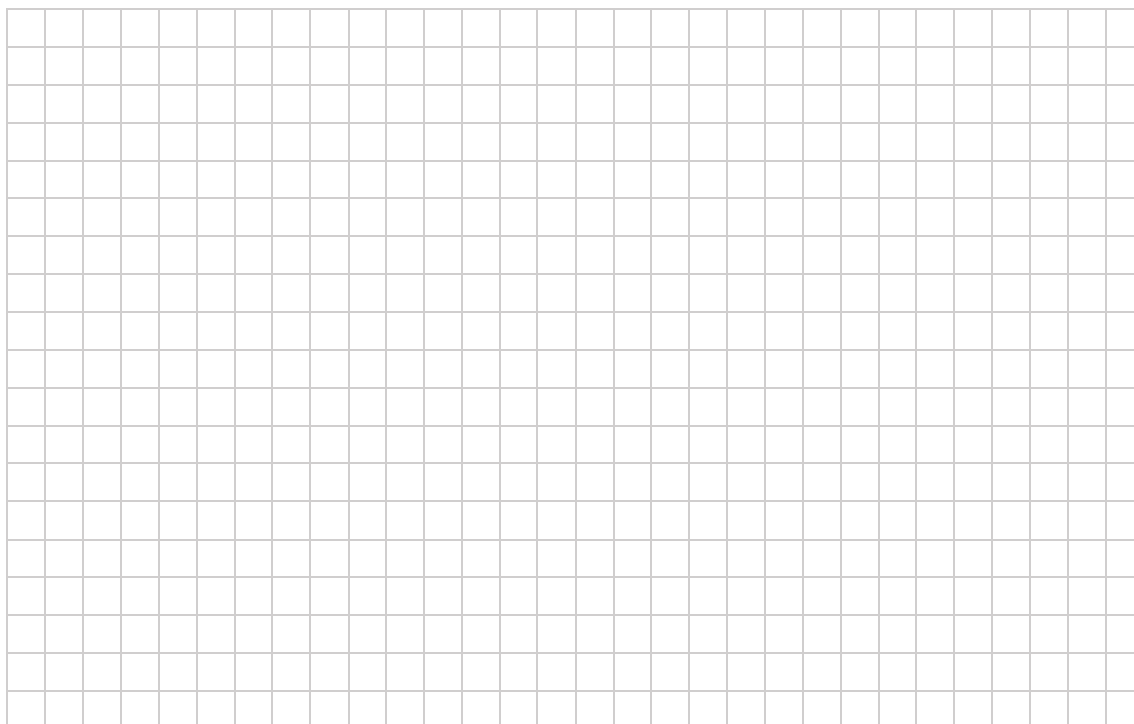
Draw and shade suitable *wholes* that represent:

a) nine-fifths

b) seven-thirds

c) $\frac{25}{9}$

d) $\frac{57}{6}$



For example, $2\frac{2}{5}$ has a whole number part of 2 and a fractional part of $\frac{2}{5}$.

- the fractional part is in simplest form *and*
- the fractional part is bigger than zero and less than 1.

Write each mixed number in simplest form, if not already so.

a) $3\frac{2}{4}$

c) $11\frac{7}{9}$

e) $9\frac{6}{8}$

b) $5\frac{3}{12}$

d) $7\frac{4}{16}$

f) $8\frac{10}{15}$

[illegible]

Write each mixed number in simplest form, if not already so.

a) $3\frac{5}{4}$

c) $4\frac{7}{5}$

e) $7\frac{14}{4}$

b) $5\frac{4}{3}$

d) $10\frac{9}{4}$

f) $8\frac{27}{12}$

[illegible]

Question 5

Convert each improper fraction to a mixed number in simplest form.

a) $\frac{9}{2}$

e) $\frac{18}{4}$

i) $\frac{62}{8}$

b) $\frac{32}{3}$

f) $\frac{28}{4}$

j) $\frac{57}{9}$

c) $\frac{43}{5}$

g) $\frac{16}{6}$

k) $\frac{49}{12}$

d) $\frac{54}{10}$

h) $\frac{59}{6}$

l) $\frac{104}{12}$

A large grid of graph paper with two vertical lines drawn through it, creating three columns for working space.

10. Jamie wondered ...

Question 1

After completing the previous section, Jamie wondered about $\frac{0}{5}$.

“Is $\frac{0}{5}$ a fraction?”, “If $\frac{0}{5}$ is a fraction, what does it represent?”, Jamie wondered.

Do your own wondering and then have a chat with some other people about $\frac{0}{5}$.

What did you conclude about $\frac{0}{5}$?

Question 2

Jamie also wondered about $\frac{5}{0}$.

Do your own wondering and have a chat with some other people about $\frac{5}{0}$.

What did you conclude about $\frac{5}{0}$?

Question 3

Consider $\frac{7}{\frac{1}{2}}$.

What does it represent?