The In-betweens

Not-whole numbers and their various forms

Booklet 2 (of 5)

In this booklet:

- **Types of fractions**
- **Mixed numbers and improper fractions**
- **Types of numbers**
- **Fractions on a number line**
- **The decimal form of a fraction**
- **Ranking numbers in decimal form**

Student Name: ____

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Smooth Mathematics

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The In-Betweens
Not-whole numbers and their forms

Booklet 2 (of 5)

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10. Types of fractions

We call the family of numbers, 1, 2, 3, 4, …, the *whole numbers*.
They are part of a bigger family of numbers, …-3, -2, -1, 0, 1, 2, 3 …, called the *integers*.
Some of the numbers in-between the integers are called fractions.
There are different types of fractions.

A *vulgar* fraction (also known as *simple* or *common*) is a number written in the form $\frac{a}{b}$ where:

- a is an *integer* and called the *numerator*,
- b is a non-zero *integer* and called the *denominator* and
- the horizontal line is called the *fraction bar* or *division bar*.

The word vulgar might seem a strange choice. But in ancient times it meant *common*.
Numbers like $\frac{2}{3}$ were in *common* use and so became known as vulgar fractions.

There are different types of vulgar fractions.

If $a < b$ and $a \neq 0$, for example $\frac{3}{4}$ or $\frac{23}{41}$, the fraction is called a proper fraction.

Proper fractions are greater than *nothing* and less than *a whole*.
Or numerically, greater than zero and less than 1.

If $a \geq b$, for example $\frac{7}{5}$ or $\frac{32}{11}$ or $\frac{10}{10}$ or $\frac{5}{1}$, the fraction is called an improper fraction.

Improper fractions are greater than or equal to *a whole*.
Or numerically, greater than or equal to 1.

If $a = 1$, for example $\frac{1}{9}$, the fraction is called a unit fraction.

Question 1

Correctly classify each number as one or more of the following: a *proper* fraction, an *improper* fraction, a *unit* fraction, a *mixed number* or "none-of-these".

a) $\frac{11}{7}$ c) $\frac{5}{9}$ e) $\frac{1}{241}$ g) $\frac{21}{9}$ i) $\frac{9}{9}$

b) $\frac{1}{12}$ d) $12\frac{1}{6}$ f) $\frac{4}{17}$ h) $\frac{24}{12}$ j) $3\frac{14}{7}$

Question 2

How many proper fractions, in simplest form, have a denominator of:

a) 5 b) 10 c) 15 d) 20 e) 100

Question 3

There exist different types of fractions other than those you have already met.
Fractions that are *not* commonly in use, so they are not classed as vulgar fractions.
For now, we will call them *mischievous* fractions.

One example of a mischievous fraction is $\frac{4\frac{1}{3}}{6}$ (four-and-one-third sixths).

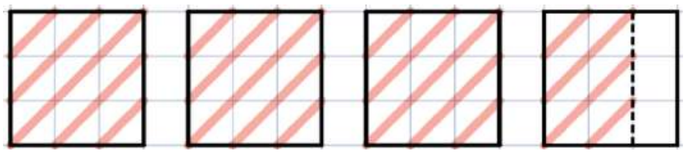
Can you reason why $\frac{4\frac{1}{3}}{6}$ is less than 1? In fact, $\frac{4\frac{1}{3}}{6} = \frac{13}{18}$. True!

Do your best to figure out, without a calculator, the vulgar fraction that is equal to each mischievous fraction.
A calculator will give you the answer but may not help you to reason why the answer is what it is.

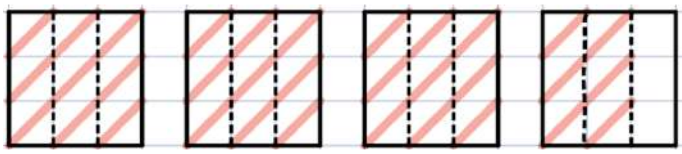
a) $\frac{2\frac{1}{8}}{3}$ b) $\frac{3\frac{1}{4}}{5}$ c) $\frac{3\frac{1}{3}}{2}$ d) $\frac{\frac{1}{2}}{\frac{1}{3}}$ e) $\frac{\frac{1}{5}}{\frac{1}{4}}$ f) $\frac{1}{2\frac{1}{5}}$ (😳)

11. Mixed numbers to improper fractions

The mixed number $3\frac{2}{3}$ can be represented with this diagram:



To convert $3\frac{2}{3}$ to an improper fraction, we can divide the *three wholes* into $\frac{1}{3}$s like this:



So, in total we have *eleven* $\frac{1}{3}$s (3×3+2) which we can write as $\frac{11}{3}$

Note that each of the wholes were divided equally into *the denominator's number* of pieces. Doing this allows us to count of how many $\frac{1}{3}$s there are in total.

In this case we have *nine* $\frac{1}{3}$s from the wholes, and the *two extra* $\frac{1}{3}$s from the non-whole.

To convert $5\frac{4}{7}$ to an improper fraction we can:

- divide each of the 5 wholes into $\frac{1}{7}$s, which gives $7 \times 5 = 35$ sevenths
- add on the *extra* 4 sevenths, $35 + 4 = 39$
- so, $5\frac{4}{7} = \frac{39}{7}$

Question 1

Convert each of the following to improper fractions in simplest form.

a) $3\frac{2}{7}$ b) $5\frac{5}{6}$ c) $7\frac{3}{8}$ d) $8\frac{2}{9}$ e) $4\frac{9}{7}$ f) $3\frac{2\frac{2}{3}}{6}$

12. Types of numbers and how we choose to represent them

Before we go too much further, it would be nice to be sure you have a good understanding of what it means for a number to:

- have a *name*,
- be of a certain *type* and
- be written in a certain (symbolic) *form*.

You might already have a good understanding of these ideas.
If you do, the table below and the information below it will make sense to you.

Name	**Type**	**Decimal form**	**Fractional form**	**Mixed number form**
four	whole	4	$\frac{4}{1}=\frac{8}{2}=\frac{12}{3}=\ldots$	–
two hundred and fourteen	whole	214	$\frac{214}{1}=\frac{428}{2}=\frac{642}{3}=\ldots$	–
one-quarter	not-whole	0.25	$\frac{1}{4}=\frac{2}{8}=\ldots$	–
two-and-one-half	not-whole	2.5	$\frac{5}{2}=\frac{10}{4}=\ldots$	$2\frac{1}{2}$

Zero, one, two, three, four, … are called the set of *whole numbers**.
*Whole numbers are part of a larger family of numbers called *integers*.

The number *two hundred and fourteen*, when written in *decimal form*, is 214, using the *digits* 1, 2 and 4.
Two hundred and fourteen can also be written in *place value form* as:

$$2 \text{ groups of } 100 + 1 \text{ group of } 10 + 4 \text{ groups of } 1$$
or
$$(2 \times 100) + (1 \times 10) + (4 \times 1)$$

We will call numbers *in-between* the whole numbers the *not-whole numbers*.

Not-whole numbers can be written in *word form*, *decimal form* and *fractional form*.*

Whole numbers can also be written in *word form*, *decimal form* and *fractional form*.

* This is not quite true for some strange not-whole numbers that you will meet in the future.

Not-whole numbers have a *whole part* and a *fractional part*.
For example,

- the whole part of $5\frac{1}{3}$ is 5 and the fractional part is $\frac{1}{3}$.
- the whole part of 0.5 is 0 and the fractional part is $\frac{1}{2}$ or 0.5.

If what is written above pretty much makes sense, you might like to skip to the questions in this section.

If not, or anyway, the story on the next three pages will most likely help.
Please give it a read. ☺

12.1 The Story

Almost every day, every person asks at least one of the following questions:

- How many … ?
- How much …?
- What is the order … ?
- What is your pin number/post code/ …?

Humans often need to count things, measure things, rank things or label things and we do this using *numbers*.

You already know a lot about numbers and can do many things with them.
This section aims to clarify and introduce some terms so that new learning is easier to discuss.

The most fundamental idea that underpins counting *whole identical objects* is the idea that the *next amount* is always *a single whole bigger*, or what you know as **+1**.

Let's say we are counting vertical sticks (but it could be anything).

| next | | next | | | next | | | | next | | | | |

The + and the 1, in +1, are *symbols* that, together, represent the idea of *a single whole more*.

Mmm, getting a little deep here, I know. But stick with me.

Think about these ideas,

- the smallest number of vertical sticks (VSs)
- the next largest number of VSs
- the next largest number of VSs
- …

Humans created both *names* and *symbols* to represent these ideas.

idea	name	Symbol
smallest number	zero	0
next largest number	one	1
next largest number	two	2
next largest number	three	3
…		

Old news?
Do you remember this story from an earlier booklet?
I hope you do, as it will not be told in full here, we will speed up a little now. ☺

Numbers we use to count things are called the set of *natural numbers*. Long ago, this set of numbers were given a special shorthand symbol, $\mathbb{N}$ (called the double-struck N).

The natural numbers might have been introduced to you as *whole numbers* (because they count whole objects) and so we will refer to them as whole numbers. It is a nice descriptive name.

There is an endless list of *whole numbers*, endless! Each one larger than the previous.
So that means an endless list of names and symbols. 😳
Who could remember such a thing?
Luckily there was no need too, because a clever *system* was created, named the *decimal system*. You most likely know a lot about this system.

The decimal system has only *decem* (ten) symbols: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
These symbols are called *digits*.
In turn, each digit represents:

$$\_,\ |,\ ||,\ |||,\ ||||,\ |||||,\ ||||||,\ |||||||,\ ||||||||,\ |||||||||$$

But what comes next?

This is the lovely bit. Reuse 1, but *not* to represent a single whole (|).
But to represent the next amount, which is ||||||||||.
10, represents *one* group of ten and zero ones (also called units).

What next?
$11, 12, 13, 14, \ldots, 20, 21, \ldots, 30, \ldots, 90, \ldots, 99, 100, \ldots, 1000, \ldots, 10000, \ldots$
Familiar?

So, 7438 represents the number

$$||||||||||||||||||||||||||||||||||||||\ldots \text{ (too many ☺)},$$

in the following way,

$$7 \text{ groups of } 1000 + 4 \text{ groups of } 100 + 3 \text{ groups of } 10 + 8 \text{ groups of } 1$$

or

$$(7 \times 1000) + (4 \times 100) + (3 \times 10) + (8 \times 1)$$

7438 is called the *decimal representation* of the number.
We often say 7438 is the *decimal form* of the number.

$(7 \times 1000) + (4 \times 100) + (3 \times 10) + (8 \times 1)$ is called the *place value form* of the number.

You have met a whole bunch of numbers that are *not whole numbers*.
We might think of them as the *in-betweens*! ☺
For example,

$$\frac{1}{2}, \frac{3}{4}, \frac{1}{5}, \frac{9}{4}, \frac{23}{9}, \frac{7}{10}, \frac{4}{100}, \text{ and } \frac{9}{1000}.$$

Or,

$$2\frac{1}{4}, 5\frac{1}{3}, 12\frac{1}{6}$$

These *in-between* numbers have a technical name, but we want to be sure you understand what they are and so for now we will call them *not-whole numbers*.

Not-whole numbers can be written in *fractional form* or *mixed number form*.
You have already been working with both these forms.

Numbers in fractional form are commonly called *fractions*.
Vulgar fractions are the most common fraction we see in life; so much so, that the word vulgar is rarely used when talking about (vulgar) fractions.

Numbers in mixed number form are commonly called *mixed numbers*.

Not-whole numbers can also be written in decimal form.
You probably know that:

$$0.5 = \frac{1}{2}$$

$$2.25 = 2\frac{1}{4} = \frac{9}{4}$$

You will learn more about not-whole numbers in decimal form in the section after next.

Not-whole numbers like 2.25 are commonly, *and wrongly*, called decimals.
Strictly speaking they are *not-whole numbers* written *in decimal form*.
Maybe you can see why they are commonly called decimals. ☺

Not-whole numbers have two parts: a whole part and a fractional part.

The whole part of $5\frac{1}{3}$ is 5 and the fractional part is $\frac{1}{3}$.

The whole part of 0.5 is 0 and the fractional part is $\frac{1}{2}$, or 0.5.

Lastly, whole numbers can be written in fractional form too!
For example,

$$12 = \frac{12}{1} = \frac{24}{2} = \frac{36}{3} = \ldots$$

What type of fraction is $\frac{24}{2}$? 🤔

Question 1

Classify each of the following as being either:

1. a whole number, written in decimal form
2. a whole number, written in fractional form
3. a not-whole number, written in decimal form
4. a not-whole number, written in fractional form

a) $\frac{12}{4}$ b) 2564 c) $\frac{5}{6}$ d) 234.23 e) $\frac{19}{12}$ f) 23.0

Question 2

Write the number seven-and-three-quarters ($7\frac{3}{4}$) in both fractional form and decimal form.

Question 3

Using the digits 2, 5, and 7 at most once, write down as many *not-whole numbers* as you can, in *decimal form*.

Question 4

Complete the following table.

Name	**Type**	**Decimal form**	**Fractional form**	**Mixed number form**
				$3\frac{1}{4}$
	whole			
			$\frac{32}{2} = \frac{48}{3} = \dots$	
one-fifth				

13. Fractions on the number line

I can just as easily eat $\frac{1}{4}$ of a delicious cake, as I can eat 2 cakes. 😌

I can just as easily vertical-jump $\frac{3}{4}$ of a metre as I can 3 metres. 😂

I can see that $\frac{1}{2}$ of one metre is less than $\frac{3}{4}$ of one metre.

Instead of thinking about cake or metres or … – if I just think about *the whole* as the number 1, then we can happily place fractions on a number line.

In this section when we mention a fraction, like $\frac{6}{7}$ or $\frac{10}{3}$, you can think of it as $\frac{6}{7}$ of 1 or $\frac{10}{3}$ of 1; but we will not write the *of 1.*

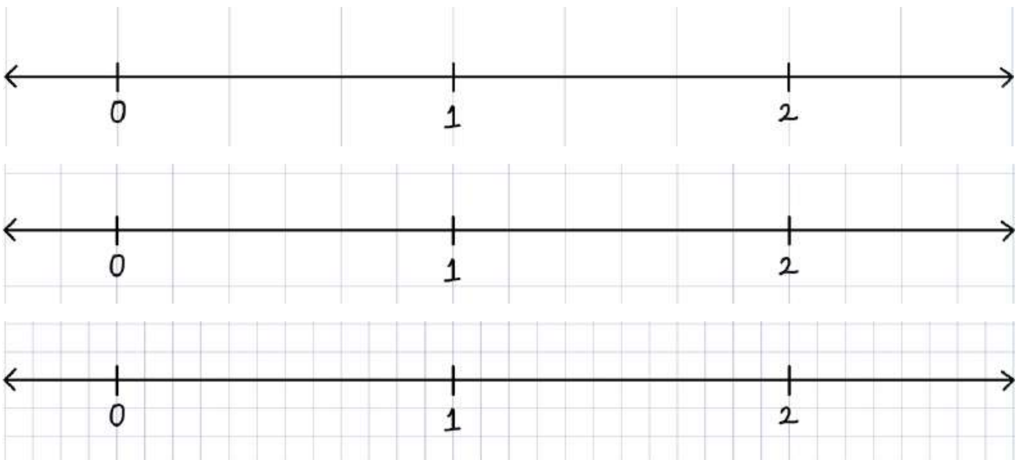
So, if we can put fractions on a number line, we can start to think about them similarly to how we think about whole numbers, since 5 is *five (lots) of 1.*

Fractions are *numbers*. ☺

Question 1

Mark each of the following fractions on *just one* of the number lines below.

a) $\frac{3}{4}$ b) $\frac{2}{3}$ c) $\frac{5}{6}$ d) $\frac{7}{12}$ e) $\frac{19}{12}$ f) $\frac{3}{2}$



Question 2

Write these fractions (taken from question 1) in order of size from smallest to largest.

$$\frac{3}{4}, \frac{3}{2}, \frac{19}{12}, \frac{2}{3}, \frac{7}{12}, \frac{5}{6}$$

Question 3

Draw your own number line and mark each of the following fractions it.

a) $\frac{1}{3}$ b) $\frac{1}{6}$ c) $\frac{2}{3}$ d) $\frac{5}{6}$ e) $\frac{6}{4}$ f) $\frac{11}{6}$

Question 4

Draw your own number line and mark each of the following fractions it.

a) $\frac{2}{3}$ b) $\frac{3}{5}$ c) $\frac{8}{15}$ d) $\frac{8}{5}$ e) $\frac{4}{3}$ f) $\frac{200}{101}$

Question 5

Write these fractions in order of size from smallest to largest.

$$\frac{1}{9}, \frac{1}{57}, \frac{1}{7}, \frac{1}{15}, \frac{1}{4}, \frac{1}{6}, \frac{1}{101}, \frac{1}{12}, \frac{1}{22}$$

Question 6

a) What is the largest unit fraction?

b) What is the smallest improper fraction?

Question 7

What is the smallest unit fraction?

Question 8

What is the largest proper fraction?

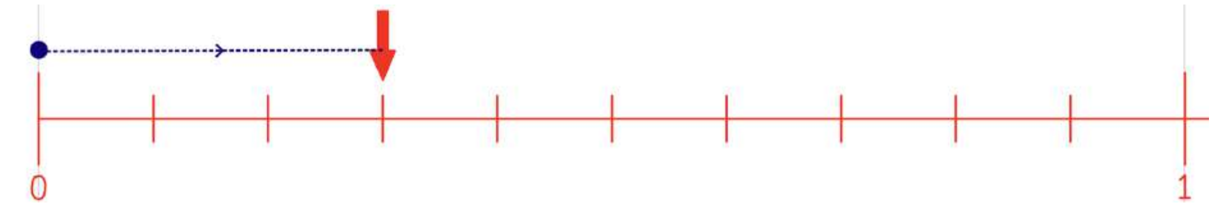
14. Decimal form for not-whole numbers – explained

You probably realised, long ago, that the number line acts as a tool for measurement. Rulers and tape measures are just special number lines.

On a ruler, the 1 is not just the *number* 1 nor is it 1 *general unit*. Rather, it is 1 specific unit – like a metre or inch or …

Shown below is the *start* of a long ruler that measures *distance* in the very rarely used unit called the *pinocchio* (shortened to *pch* 😟).

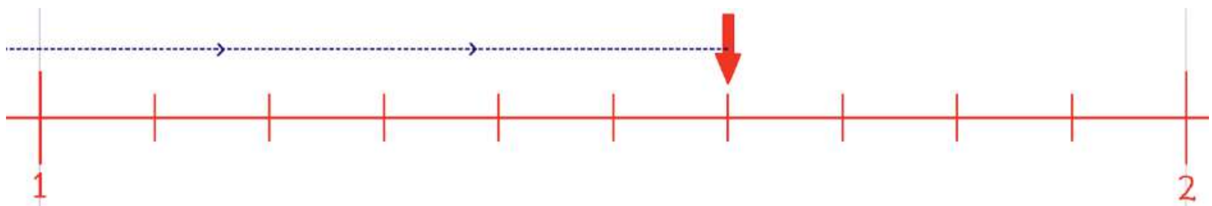
The red arrow, seen immediately below, moved from the start (0) and stopped where you see it.



The red arrow has moved

$$\frac{3}{10}\text{ pch.}$$

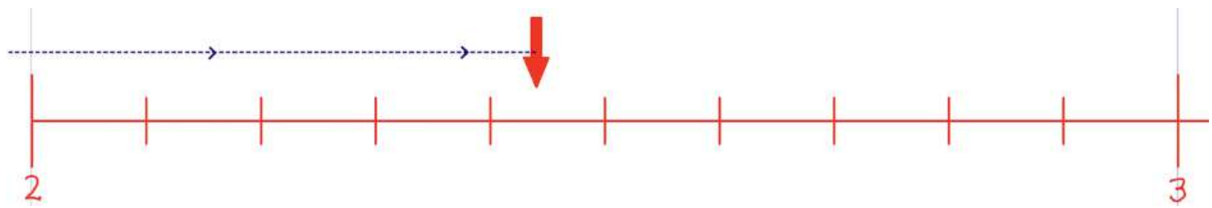
Shown below is a section of the ruler from 1 pch to 2 pch.
The red arrow started again and stopped where you see it.



It has now moved a total of

$$1 + \frac{6}{10}\text{ pch.}$$

The arrow kept moving forwards, past 2, and stopped again where it is seen below.



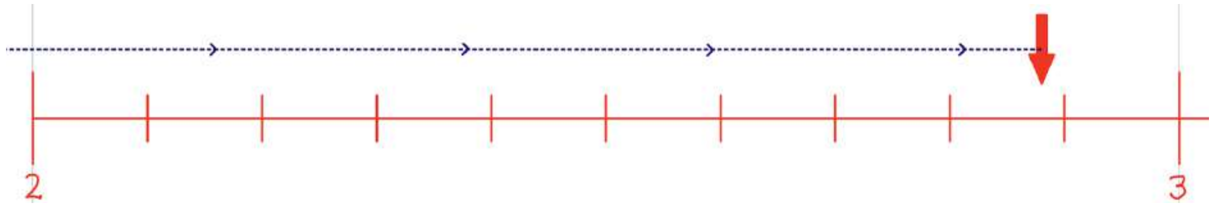
It is difficult to be accurate in stating how far the red arrow has now travelled, so the best we can say is the red arrow has now moved

$$\textit{more than } 2 + \frac{4}{10}\text{ pch and } \textit{less than } 2 + \frac{5}{10}\text{ pch.}$$

A fancy way of writing this fact is,

$$2 + \frac{4}{10}\text{ pch} < \text{distance travelled} < 2 + \frac{5}{10}\text{ pch}$$

There is no stopping this red arrow!
It kept moving forwards and then stopped where you see it, immediately below.



Again, it is difficult to be accurate, so the best we can say is the red arrow has now moved

$$\textit{more than } 2 + \frac{8}{10} \text{ pch and } \textit{less than } 2 + \frac{9}{10} \text{ pch.}$$

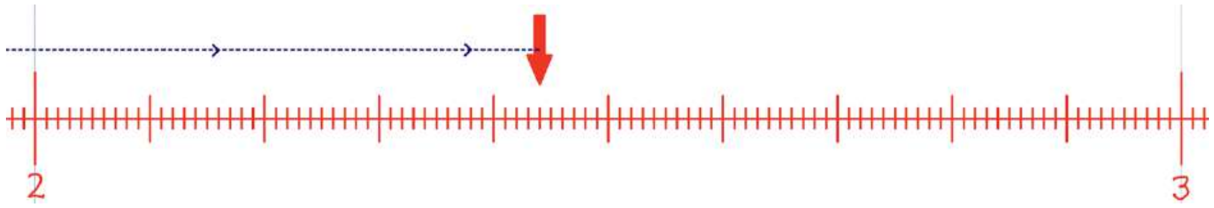
How could we be more accurate with our measurements in the last two cases? 🤔

We could sub-divide the $\frac{1}{10}$s into smaller divisions; but how many?

If ten divisions were good enough between the whole numbers, 1 and 2, 2 and 3, …, why not use ten again?

So, each 10 sub-divisions of 1 pch will have 10 *smaller* (but equal) sub-divisions.

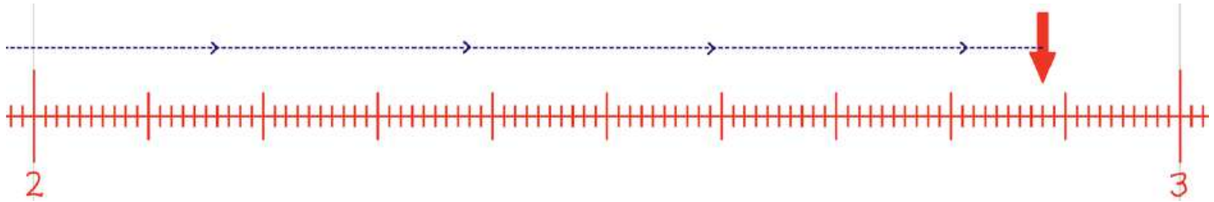
Thus, each of the smaller divisions will be $\frac{1}{100}$ pch.



So now, looking at the red arrow's first stopping point in between 2 and 3 (seen immediately above), we can now see it travelled *exactly*

$$2 + \frac{4}{10} + \frac{4}{100} \text{ pch}$$

Now let's look again at the red arrow's second stopping point between 2 and 3.



We can now see it travelled *exactly*

$$2 + \frac{8}{10} + \frac{8}{100} \text{ pch}$$

But.
What if the red arrow stopped a teeny-tiny bit to the right of where it last stopped, just past the $\frac{8}{100}$ mark and just before the $\frac{9}{100}$ mark?

Mmmm. 🤔

So, the travel-some red arrow started at zero and made stops at:

$$\frac{3}{10}\text{ pch, } 1+\frac{6}{10}\text{ pch, } 2+\frac{4}{10}+\frac{4}{100}\text{ pch and } 2+\frac{8}{10}+\frac{8}{100}\text{ pch}$$

You might be wondering a couple of things, like:

- Why did we not simplify those fractions?
- Why did we include the + sign and not just write $1\frac{6}{10}$ or $\frac{16}{10}$ or $\frac{8}{5}$?

We did that so you could see, or be reminded of, how the **decimal system** extends to *not-whole* numbers.

$$1+\frac{6}{10}\text{ pch can be written as } 1.6\text{ pch}$$

$$2+\frac{4}{10}+\frac{4}{100}\text{ pch} = 2.44\text{ pch}$$

$$\frac{3}{10}\text{ pch} = 0.3\text{ pch}$$

Every number that can be written as a fraction can also be written in *decimal form*.

Decimal form is a short-cut way of writing a number in terms of the ***sum*** of its

$$\ldots\ 1000\text{s, } 100\text{s, } 10\text{s, } 1\text{s, } \frac{1}{10}\text{s, } \frac{1}{100}\text{s, } \frac{1}{1000}\text{s, } \ldots$$

The *decimal point* separates the place values of *1 and greater*, from those that are less than 1.

If a faction has a denominator of 10, or 100, or 1000 or … (increasing powers of ten) then it is called a *decimal fraction* and it is straightforward to write it in *decimal form*.
For example:

$$\frac{7}{10} = 0.7$$

$$\frac{72}{100} = \frac{70}{100}+\frac{2}{100} = \frac{7}{10}+\frac{2}{100} = 0.72$$

$$\frac{5701}{10000} = \frac{5000}{10000}+\frac{700}{10000}+\frac{0}{10000}+\frac{1}{10000} = \frac{5}{10}+\frac{7}{100}+\frac{0}{1000}+\frac{1}{10000} = 0.5701$$

$$\frac{743}{100} = \frac{700}{100}+\frac{40}{100}+\frac{3}{100} = 7+\frac{4}{10}+\frac{3}{100} = 7.43$$

$$\frac{51}{1000} = \frac{50}{1000}+\frac{1}{1000} = \frac{\mathbf{0}}{\mathbf{10}}+\frac{5}{100}+\frac{1}{1000} = 0.051$$

A 0 before the decimal point tells us there is zero 1s (or units) *and* zero of every higher place value.

Question 1

Write down the *decimal form* of each of the following fractions.

a) $\frac{2}{10}$ b) $\frac{25}{100}$ c) $\frac{358}{1000}$ d) $\frac{4509}{10000}$ e) $\frac{304}{1000}$ f) $\frac{8001}{10000}$

Question 2

Write down the *decimal form* of each of the following numbers.

a) $\frac{61}{1000}$ b) $\frac{8}{10000}$ c) $\frac{204}{100}$ d) $\frac{4231}{1000}$ e) $31\frac{21}{1000}$ f) $11\frac{1}{10000}$

15. From *fractional form* to *decimal form*

15.1 Fractions with *easy* denominators

Earlier we said every fraction can be written in decimal form. But what if the denominator is not 10, or 100, or 1000 or …? I.e. what if they are not *decimal fractions*.

For example, $\frac{3}{4}$?

You probably *know* this one, but let's think through a process that will work for cases you do not already know.

We could change $\frac{3}{4}$ to an equivalent (equal) fraction with denominator 100 by multiplying both the numerator and denominator by 25.

So,

$$\frac{3}{4} = \frac{3\times 25}{4\times 25} = \frac{75}{100} = \frac{70}{100} + \frac{5}{100} = \frac{7}{10} + \frac{5}{100} = 0.75$$

Another example. What is the decimal form of $\frac{1}{20}$?

$$\frac{1}{20} = \frac{5}{100} = 0.05$$

Question 1
Write each of the following fractions in decimal form.

a) $\frac{31}{50}$ b) $\frac{63}{50}$ c) $\frac{39}{25}$ d) $\frac{46}{25}$ e) $\frac{17}{20}$ f) $\frac{5}{4}$

Question 2

Write each of the following numbers in decimal form.

a) $3\frac{21}{50}$ b) $\frac{33}{20}$ c) $\frac{101}{50}$ d) $5\frac{43}{50}$ e) $34\frac{2}{25}$ f) $43\frac{16}{25}$

15.2 Fractions with *not-so-easy* denominators

Consider the fraction $\frac{5}{6}$. What is its decimal form?

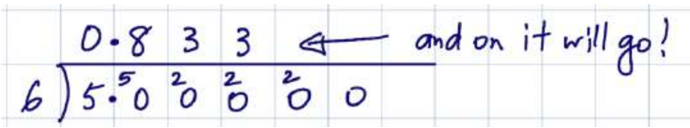
6 does not easily multiply up to 10 or 100 or …, so what to do?

Recall that we can think about $\frac{5}{6}$ as $5 \div 6$.

We can use this idea to write *any fraction* in decimal form, in this case by dividing 5 by 6.

We can do the division using an *extension* of either the short or long division algorithm for whole numbers.
Either can be extended by tacking on *a decimal point and some 0s* to the *dividend*, as we know that place values less than one will be required.

Here we go.
Be sure to *accurately align (vertically)* the place values and the decimal place in the dividend and the answer (quotient).



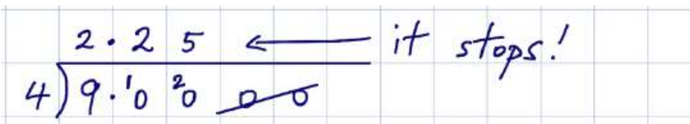
Notice '6 into 20' will just *keep coming up* and so an endless string of 3s will result.

So, $\frac{5}{6} = 0.8\overline{3}$

The bar above the 3, indicates that there will be an endless string of 3s after the first 3.
The bar is called a *vinculum*.

$0.8\overline{3}$ is said to be an *infinitely recurring decimal form*, since the 3 recurs (or repeats) forever.
$0.8\overline{3}$ is spoken as follows, *zero point eight three recurring*.

Another example.
To convert $\frac{9}{4}$ into decimal form, even though it has an *easy denominator*, we can divide as follows.



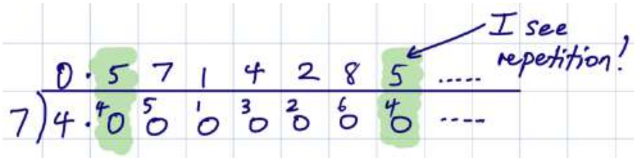
So, $\frac{9}{4} = 2.25$

2.25 is said to be a *terminating decimal form* since there are no further decimal places after the 5, as 4 divides into 20 with zero remainder.

Now, you might be wondering why that works like that. Yes?
If you would like to understand more about why this process works, please ask your teacher to show you the "cake" video. ☺

One last example.

What is $\frac{4}{7}$ written in decimal form?



So, $\frac{4}{7} = 0.\overline{571428}$

$0.\overline{571428}$ is another example of a *recurring decimal form* because the *block*, 571428, recurs, forever. There are many fractions with interesting recurring decimal forms.

Question 1

Use a division algorithm to convert each of the following fractions to decimal form.

a) $\frac{5}{8}$ c) $\frac{1}{9}$ e) $\frac{20}{7}$ g) $\frac{1}{32}$

b) $\frac{1}{6}$ d) $\frac{15}{7}$ f) $\frac{11}{9}$ h) $\frac{1}{81}$

Question 2

Classify each result from Question 1 as either *infinitely recurring* or *terminating*.

Question 3

Believe it or not, numbers exist that have a decimal form that is *infinitely non-recurring*. 😳
Some of these numbers are very famous.
Write down two such numbers that are famous and at least two that you made up yourself.
Do not write down all the digits. 😂

16. Decimal forms to remember

Some numbers in decimal form often pop up in life and it is useful to be able to recognise their fractional form.

Do your very best to memorise the following equalities.

$$\frac{1}{2} = 0.5$$

$$\frac{1}{3} = 0.\overline{3}, \frac{2}{3} = 0.\overline{6}$$

$$\frac{1}{4} = 0.25, \frac{3}{4} = 0.75$$

$$\frac{1}{5} = 0.2, \frac{2}{5} = 0.4, \frac{3}{5} = 0.6, \frac{4}{5} = 0.8$$

$$\frac{1}{8} = 0.125, \frac{3}{8} = 0.375$$

$$\frac{1}{9} = 0.\overline{1}, \frac{2}{9} = 0.\overline{2}, \text{etc.}$$

$$\frac{1}{10} = 0.1, \frac{2}{10} = 0.2, \text{etc.}$$

$$\frac{1}{50} = 0.02, \frac{2}{50} = 0.04, \text{etc.}$$

$$\frac{1}{100} = 0.01, \frac{2}{100} = 0.02, \text{etc.}$$

Question 1

Carry out *two* ‘different’ calculation processes which verify $\frac{1}{8} = 0.125$.

17. Ranking numbers in decimal form

Which of these whole numbers is larger, 4000 or 3999?
You know the answer, of course, but there is a point to the question.

Even with all those zeros in 4000, 4000 is larger than 3999 (with all its nines).
But why?
Well, it is because of how the decimal system works.
999 represents the number immediately before (or 1 less than) 1000 and so it is

impossible for any 3-digit number to be larger than a 4-digit number.

In this case, since $3 < 4$, 4000 is the larger because it has more thousands.

> So, to determine which of two whole numbers is the larger, do the following:
>
> - Compare the digits of the largest place value. The number with the largest digit in that place is the largest number.
> - If the digits in the largest place value are equal, move right, to the next place value and compare the digits, the number with the largest digit is largest number.
> - If the digits are equal, move right, to the next place value …
> - *Keep going until you run out of place values.*

But does this process work if we wanted to compare

32.1450892359999999 and 32.1451?

Yes, it does! ☺
But why?

Consider the two numbers 0.1 and 0.09999999999999999999
The first digit after the decimal point represents how many $\frac{1}{10}$s the number has.
The second digit after the decimal point represents how many $\frac{1}{100}$s the number has.
So, even if the first digit after the decimal point is as small as it can be, above zero, in the first number (i.e., 1) and the second digit is as large as it can be in the second number (i.e., 9), we are comparing $\frac{1}{10}$ to $\frac{9}{100}$s and since

$$\frac{1}{10} = \frac{10}{100}, \frac{1}{10} \text{ is larger.}$$

But won't all the rest of the 9s change the situation? No!

If you take some time to think carefully about it, you will be able to reason that each successive 9 is representing an increasingly smaller amount and the sum of the amounts will never reach $\frac{1}{10}$.
You will get a chance to think about this further in the question set.

The fact that all those 9s will not 'overtake' the 1, means you can use the same process to rank whole numbers and not-whole numbers (in decimal form). ☺

Example 1.

Which is larger, 0.10056 or 0.1008

0.10056
0.10080

$8 > 5$

$\Rightarrow$ 0.1008 is larger

Example 2.

Which is larger, 27.243 or 27.19999

27.24300
27.19999

$2 > 1$

$\Rightarrow$ 27.243 is larger

Example 3.

Which is larger, 0.461 or 0.48999

0.46100
0.48999

$8 > 6$

$\Rightarrow$ 0.48999 is larger

Example 4.

Which is larger, $\frac{123}{240}$ or $\frac{217}{430}$.

Using my calculator to find the decimal form of the fraction:

$\frac{123}{240} = 0.5125$

$\frac{217}{430} = 0.5046.....$

$\therefore \frac{123}{240}$ is larger.

Question 1

Determine which of the numbers in each pair is the largest.

a) $0.8915, 0.8909$

b) $11.8945, 11.8954$

c) $9.2345, 9.234$

d) $3.4005, 3.405$

e) $0.00499, 0.0005$

f) $27.00002, 27.000019$

Question 2

Determine which of the numbers in each pair is the largest.

a) $\frac{52}{105}, \frac{71}{143}$

b) $6.9519485\ , \frac{3761}{541}$

c) $\frac{3464}{56721}, \frac{6928}{113443}$

Question 3

Use either $<$, $=$ or $>$ to correctly indicate the size of one number compared to the other.

a) $\frac{34}{7}$, 4.857142857

b) $\frac{253}{12}$, 21.08333333

c) $\frac{71}{6}$, $11.8\overline{3}$

Question 4

Draw a section of the number line between 32 and 33 and accurately mark the following numbers on the number line.

32, 32.8, 32.35, 32.04, 32.092, 33

Question 5

Draw a section of the number line between 32 and 32.5 and accurately mark the following numbers on the number line.

32, 32.04, 32.355, 32.44, 32.092, 32.5

Question 6

Do your best to explain why 0.0999999999999999999999, with as many 9s as you wish to write, is not larger than 0.1.

18. From *decimal form* to *fractional form*

We now understand the decimal form of a number is simply a notation to represent fractional form more efficiently. For example,

$$\frac{7}{10} = 0.7$$

$$\frac{72}{100} = \frac{70}{100} + \frac{2}{100} = \frac{7}{10} + \frac{2}{100} = 0.72$$

$$\frac{5701}{10000} = \frac{5000}{10000} + \frac{700}{10000} + \frac{0}{10000} + \frac{1}{10000} = \frac{5}{10} + \frac{7}{100} + \frac{0}{1000} + \frac{1}{10000} = 0.5701$$

$$\frac{743}{100} = \frac{700}{100} + \frac{40}{100} + \frac{3}{100} = 7 + \frac{4}{10} + \frac{3}{100} = 7.43$$

$$\frac{51}{1000} = \frac{50}{1000} + \frac{1}{1000} = \frac{\mathbf{0}}{\mathbf{10}} + \frac{5}{100} + \frac{1}{1000} = 0.051$$

To change a number in decimal form to fractional form, we note the *place value* of the right-most digit and this becomes the *base* fraction with which we start.

For example, in the number 0.62 the digit 2 has place value of $\frac{1}{100}$ so,

$$0.62 \text{ represents } \frac{2}{100} \text{ plus } \frac{60}{100}, \text{ so } \frac{62}{100} \text{ in total.}$$

We then check to see if $\frac{62}{100}$ can be simplified, and in this case $\frac{62}{100} = \frac{31}{50}$. So, $0.62 = \frac{31}{50}$.

In short,

- the *numerator* of the fraction is the number in decimal form written **without** the decimal point and
- the *denominator* is the place value of the right-most digit.

Example 1.

Write 0.24 as a fraction in simplest form.

$$0.24 = \frac{24}{100} = \frac{12}{50} = \frac{6}{25}$$

Example 2.

Write 0.402 as a fraction in simplest form.

$$0.402 = \frac{402}{1000} = \frac{201}{500}$$

Example 3.

Write 0.28 as a fraction in simplest form.

$$0.0028 = \frac{28}{10000} = \frac{14}{5000} = \frac{7}{2500}$$

Example 4.

Write 2.04 as a mixed number and fraction in simplest form.

$$2.04 = 2\frac{4}{100} = 2\frac{1}{25} = \frac{51}{25}$$

Question 1

Write each number as a fraction in simplest form.

a) 0.75

b) 0.4

c) 0.125

d) 0.44

e) 0.034

f) 0.0202

g) 214

h) $0.\overline{25}$

Question 2

Write each number as a mixed number and fraction in simplest form.

a) 4.25

b) 1.02

c) 2.0325

d) 2.0404

e) 200.001

f) 1.23

g) 1.2345

h) $2.\overline{101}$