The In-betweens

Not-whole numbers and their various forms

Booklet 3 (of 5)

In this booklet:

- **The percentage form of a fraction**
- **Percentage of a quantity**
- **One quantity as a percentage of another**
- **Comparing the size of fractions**
- **LCD (LCM)**
- **Adding and subtracting fractions**

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Smooth Mathematics

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Not-whole numbers and their forms

Booklet 3 (of 5)

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Contents

19. Fractions as percentage

19.1 The Romans

In ancient Rome (753 BC – 476 AD, a long time ago) life was both very different and quite similar to life today. One similarity was taxation.

A tax is an amount of money that *civilians* (the people) pay to the '*state*' (the government).

In ancient Rome, the people paid taxes for various things. Examples include, tax for:

- selling things at auction (including slaves)
- freeing a slave,
- receiving an inheritance,
- being an unmarried man.

One way to pay for things was with a coin called the *denarius*.
Two examples of how much things cost are:

- a cow, the price of which varied around 200 denarii,
- a male slave, the price of which varied around 500 denarii.

Notice these values are in the hundreds.
Many other things, regularly for sale, would have had similar prices.

> In ancient Rome, if an item was sold at auction the seller had to pay a tax, of *one-hundredth of the selling price* $\left(\frac{1}{100}\right)$, to the state (government).
>
> In hard times, like wartime, tax increased to *three-hundredths of the selling price* $\left(\frac{3}{100}\right)$.

It seems like $\frac{x}{100}$ was an easy fraction to choose, given many of the things sold were in the hundreds of dollars. $\frac{x}{100}$ is often called the *tax rate*.

Suppose it was wartime in ancient Rome, and a cow was sold for 340 denarii.
How much tax would the seller have paid?

$$\begin{aligned}\text{tax} &= \frac{3}{100} \text{ of } 340 \\ &= \frac{3}{100} \times 340 \\ &= \frac{1020}{100} \\ &= 10\frac{20}{100} = 10\frac{2}{10} = 10.2\end{aligned}$$

So, the tax payable would be 10.2 denarii*.

* Note that decimal form did not exist in ancient Roman times!

Question 1

Calculate the tax payable on objects sold in ancient Rome for the following prices if the tax rate is $\frac{3}{100}$.

All prices are in denarii.

a) 900 b) 150 c) 230 d) 80 e) 98 f) 2205

19.2 For every hundred – the percentage

When we calculate $\frac{3}{100}$ of 500 objects, $\frac{3}{100}$ can be thought of as,

3 for every 100.

This *for every* idea is called a rate. *For every* is often replaced with the word *per* – 3 per 100. There are five 100s (in 500) and so we require 5 lots of 3, which is 15 objects.

As time passed, the idea of using the fraction $\frac{x}{100}$ for calculating tax, stuck, and the idea was eventually given a name – ***percentage***.

$\frac{5}{100}$, for example, became known as 5 *percent* (5 for every 100).

Percentage is derived from the latin phrase *per centum*, which translates to *by hundred*. Further along in time, a symbol was used to replace the word *percent*, namely **%**.

$$\frac{7}{100} = 7 \text{ for every } 100 = 7 \text{ percent} = 7\%$$

It is critical to appreciate that a percentage is just different form of a fraction.

Every *fraction can be expressed as a percentage*, by

calculating its equivalent (equal) fraction with denominator 100.

One way to do this is to find a number that we can multiply the fraction's numerator *and* denominator by so the denominator is scaled to 100. For example:

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 75\%$$

But what about a fraction with a 'not so nice' denominator? For example, $\frac{1}{3}$; a fraction with a denominator that does not easily scale to 100.

We can start by writing an equation that describes what we want to achieve.

$$\frac{1}{3} = \frac{P}{100}$$

Now multiplying both side of this equation by 100 keeps the equation true and lets us determine P.

$$\frac{1}{3} \times 100 = \frac{P}{100} \times 100$$

$$\frac{100}{3} = P$$

$$P = 33\tfrac{1}{3}$$

So, $\frac{1}{3} = 33\frac{1}{3}\ \%$

Thanks to the process shown on the previous page, no matter what fraction we wish to express as a percentage, you can see that *multiplying the fraction by 100* will give the *percentage*.

Example 1

Express $\frac{7}{20}$ as a percentage.

——————— 0 ———————

$$\frac{7}{20} = \frac{7 \times 5}{20 \times 5} = \frac{35}{100} = 35\%$$

Example 2

Express $\frac{1}{6}$ as a percentage, P.

——————— 0 ———————

$$P = \left(\frac{1}{6} \times 100\right)\%$$

$$P = \frac{100}{6}\%$$

$$P = \frac{50}{3}\%$$

$$P = 16\frac{2}{3}\%$$

You already know a given fraction can be expressed in decimal form; each having the same value. So, a number in decimal form can also be expressed as a percentage.

To express a number in *decimal form* as a *percentage*,

we do the same as we do to a fraction *multiply the number by 100*.

Example 3

Express $\frac{1}{8}$ as a percentage, P.

——————— 0 ———————

Recall that $\frac{1}{8} = 0.125$

So, $P = 0.125 \times 100\ \%$

$P = 12.5\ \%$*

* Recall that when multiplying a number in decimal form by 100, we can move the decimal place two places to the right.

Example 4

Find the percentage (P) equal to $\frac{5}{12}$.

———————— 0 ————————

By doing a short division $\frac{5}{12} = 0.41\bar{6}$.

$$\text{So, } P = 0.41\bar{6} \times 100$$

$$P = 41.\bar{6}\ \%$$

Some percentages seem to *pop up* often and it is useful to be able to recognise their fraction and decimal form.

Do your very best to memorise the following equalities.

$\frac{1}{100} = 0.01 = 1\ \%$, $\frac{2}{100} = 0.02 = 2\ \%$

$\frac{1}{2} = 0.5 = 50\%$

$\frac{1}{10} = 0.01 = 10\ \%$, $\frac{2}{10} = 0.02 = 20\ \%$

$\frac{1}{20} = 0.05 = 5\ \%$, $\frac{3}{20} = 0.15 = 15\ \%$

$\frac{1}{3} = 0.\bar{3} = 33\frac{1}{3}\ \%$, $\frac{2}{3} = 0.\bar{6} = 66\frac{2}{3}\ \%$

$\frac{1}{4} = 0.25 = 25\ \%$, $\frac{3}{4} = 0.75 = 75\ \%$

$\frac{1}{5} = 0.2 = 20\ \%$, $\frac{2}{5} = 0.4 = 40\ \%$, $\frac{3}{5} = 0.6 = 60\ \%$, $\frac{4}{5} = 0.8 = 80\ \%$

$\frac{1}{8} = 0.125 = 12\frac{1}{2}\ \%$, $\frac{3}{8} = 0.375 = 37\frac{1}{2}\ \%$

Each row in the table above can be continued. ☺

Question 1

Express each of the following as a percentage. (All answers are whole numbers.)

a) $\frac{57}{100}$

b) $\frac{38}{50}$

c) $\frac{7}{10}$

d) $\frac{11}{20}$

e) $\frac{3}{5}$

f) $\frac{12}{10}$

g) $\frac{290}{200}$

h) $\frac{420}{500}$

Question 2

Express each of the following as a percentage, *in mixed number form*.

a) $\frac{2}{3}$

b) $\frac{3}{8}$

c) $\frac{5}{8}$

d) $\frac{1}{6}$

e) $\frac{1}{9}$

f) $\frac{7}{6}$

g) $\frac{9}{7}$

h) $\frac{7}{12}$

Question 3

Express each of the following as a percentage *in decimal form*.

a) $\frac{2}{3}$

b) $\frac{3}{8}$

c) $\frac{1}{9}$

d) $\frac{7}{12}$

e) 0.57

f) $0.45\overline{6}$

g) 0.0006

h) 0.04031

Question 4

State whether each of the following fractions is *less than* or *greater than* 50%

a) $\frac{5}{8}$

b) $\frac{5}{6}$

c) $\frac{4}{9}$

d) $\frac{3}{7}$

e) $\frac{8}{15}$

f) $\frac{13}{28}$

Question 5

State whether each fraction is closer to 25% or 50%

a) $\frac{5}{12}$

b) $\frac{11}{36}$

c) $\frac{19}{48}$

d) $\frac{7}{16}$

e) $\frac{11}{32}$

f) $\frac{3}{8}$

19.3 Percentage of a quantity

Famous quote …

The most important thing to remember about a percentage is that it is just a fraction, *written in a different form*.

A.P. Harradine, 9/2/2022 ☺

As such, finding a *percentage of a quantity* is the same as finding a *fraction of a quantity*.

As you have already learned, to find a fraction of a quantity,

we multiply the fraction by the quantity.

As the world developed and money became an important part of life, the concept of percentage became widely used in many different contexts.

Some common contexts are:

- *discount* – an amount of money removed from a sale price (a *saving* for the buyer),
- *stamp duty* – an amount of money paid to the government by the buyer of a car or land,
- *interest* – an amount of money a borrower pays on top of paying back the amount borrowed, or an amount a loaner/investor earns,
- *profit* – an amount of money added to the price of an object paid by the buyer.

Example 1.
A new fancy t-shirt is for sale at \$40, but the seller is offering a "15% off sale".
How much will I save if I purchase the t-shirt?

———————— 0 ————————

Notice the sale price is much less than \$100 but we need to find 15 for every 100.

We know that $\frac{15}{100} = \frac{3}{20}$ and so 15 per 100 is the same as 3 per 20 and so the saving will be \$3 for every \$20. Therefore, the saving is \$6.

Alternatively,

$$\begin{aligned} \text{saving } &= \frac{15}{100} \text{ of } 40 \\ &= \frac{3}{20} \times 40 \\ &= 3 \times 2 \qquad \text{since } 40 \div 20 = 2 \ \left(\text{or } \frac{120}{20}\right) \\ &= 6 \end{aligned}$$

So, I will save \$6.

Example 2.

Ashley owns a second-hand shop and always tries to make a profit of $37\frac{1}{2}\%$ on items. She buys a TV for \$240. How much profit will she try to make?

———— 0 ————

$37\frac{1}{2}\%$ is the same as $\frac{3}{8}$.

$$\text{profit} = \frac{3}{8} \times 240$$

$$= 3 \times 30 \qquad \text{since } 240 \div 8 = 30 \text{ (or } \tfrac{720}{8}\text{)}$$

$$= 90$$

So, she tries to make a profit of \$90.

Example 3.

Kim's mother offers to gift him 20% of the total amount of money he can save in the next 6 months. He saved \$435. How much did Kim receive as a gift?

———— 0 ————

20% is the same as $\frac{1}{5}$.

$$\text{profit} = \frac{1}{5} \times 432$$

$$= \frac{432}{5} \qquad \text{(doing a short division will help here)}$$

$$= 86\frac{2}{5} = 86.4$$

So, Kim is gifted \$86.40.

Example 4.

Find 76% of \$800.

$76 = 75 + 1$. 75% is the same as $\frac{3}{4}$. Amount $= \frac{3}{4} \times 800$ $= 3 \times 200$ $= 600$ Now, 1% of 800 is 8. So, 76% of \$800 = 600 + 8 = \$608.	Amount $= \frac{76}{100} \times 800$ $= \frac{19}{25} \times 800$ $= 19 \times 32$ $= 608$ So, 76% of \$800 is \$608.	1% of 800 is 8. So, 76% of 800 $= 76 \times 8$ $= 608$. So, 76% of \$800 is \$608.

Question 1

Calculate each of the following amounts.

a) 10% of 2470

b) 25% of 160

c) 60% of 45

d) 75% of 280

e) $33\frac{1}{3}\%$ of 156

f) 15% of 160

g) $12\frac{1}{2}\%$ of 6400

h) $37\frac{1}{2}\%$ of 96

i) 27% of 1600

Question 2

A school estimates that 35% of their students will want to attend a free movie night being offered. They need to organise catering (drinks, chips, …).
If the school has 820 students, for how many students do they need to cater?

Question 3

A new fishing rod normally priced at $180 is offered for sale at a "20% off sale".
How much will I saved if I purchase the fishing rod?

Question 4

RE-CYC Pty Ltd sells *recycled plastic* to manufacturers of garden irrigation systems.
Their sale price is $370 per tonne.
But they must add GST (goods and services tax) to this price. GST is 10% of the sale price.

a) How much GST is added to the sale price of $370?

b) A client buys 8 tonnes of the plastic; how much do they pay?

Question 5

Jill is keen to buy a new pair of sneakers.
The type she wants to buy are for sale in a shop for $220, and the shop is having a 15% off sale.
She also saw the same sneakers for sale, online, marked at $320, and the website was offering 40% off.

a) Which option has the larger saving?

b) Which option is cheapest?

Question 6

A massive discount of $37\frac{1}{2}\%$ is offered on the sale price of a new fridge. The sale price was $1,248. How much would a person pay if they purchased the fridge? Ignore GST.

Question 7

Which is larger?
13% of $24,000 or $68\frac{2}{3}\%$ of $4,500?

19.4 One quantity as a percentage (fraction) of another

If you ate 3 *squares* of a chocolate bar and in total the bar has 10 squares, you already know you ate $\frac{3}{10}$ of the bar.

The fraction $\frac{3}{10}$ gives us a way to *compare two quantities* – what you ate, compared to what there was to eat.

$\frac{3}{10}$ is the answer to the more *general* question,

What fraction is 3 of 10?

This section is about determining *what percentage one quantity is of another*.
Since a percentage is just a different form of a fraction, this section is actually about determining *what fraction one quantity is of another*. You already know a lot about that. ☺

Often interesting information is reported about groups of people, without the actual number of people being stated. For example:

- 78.5% of Australian children *surveyed*, aged between 5 and 14, read for pleasure. *
  (source: https://www.abs.gov.au)
- 4.9% of employable Australians were unemployed in June 2021.
  (source: https://www.abs.gov.au)
- 61% of migrant taxpayers, in Australia, held a skilled work visa in 2019.
  (source: https://www.abs.gov.au)
- Apparently, about 8% of humans have blue eyes. ($\frac{2}{25}$)
- Apparently, less than 2% of humans have red hair. ($\frac{1}{50}$) **
- Apparently, the global rate for washing one's hands after toilet use is less than 20%. ($\frac{1}{5}$)

* In the 2017-18 financial year.
** Fun fact. Typical humans are born with about 100,000 scalp hair follicles. The number varies with colour, those with red hair have the least, blondes the most.

78.5% of Australian children means 78.5 *for every hundred*. Mmm, half a child?
Clearly there was not 100 children in the survey! 😂

So, how many children could have been surveyed?
Well,

$$\frac{78.5}{100} = \frac{157}{200} = \frac{314}{400} = \frac{628}{800} = \dots$$

So, there could have been 157 out of 200, or 314 out of 400 or 628 out of 800 or …

Fractions, and hence percentages, help us to *compare two quantities*.
For example:

- the *number* who read for pleasure, compared to the *total number* surveyed,
- the *number* employed, compared to the *total number* employable,
- the *number* of times hands were washed, compared to the *total number* of toilet visits (not sure how this was measured!).

What is interesting is that when percentages are published in the news, we often do not know the exact number of people or dollars or kilograms or … – often just the percentage is quoted.

The simplified fraction, or percentage, is sometimes enough to give us the information we need, if we have an approximate idea of the actual numbers involved.

What counts as a big percent?
Greater than 50% is often considered a majority. Less than 50% a minority. But …

Suppose that 10% of people who took a certain drug experienced total hearing loss.
10% is a minority, quite small, 1 in every 10, but would you take the drug if offered?
What would affect your decision? In some cases, 10% might not be considered small.

So, how do we calculate what percentage one quantity (a) is of another quantity (b)?
We do the following:

- make a fraction of the two quantities, $\frac{a}{b}$, and then
- express the fraction as a percentage.

Example 1
Suppose that from the 180 Year 7 students in a school, 60 were in favour of *more* homework than is currently given.
What percentage of *all Year 7 students* are in favour of more homework?

0

$$\frac{60}{180} = \frac{6}{18} = \frac{1}{3} = 33\frac{1}{3}\ \%$$

Example 2
What percentage is 18 marks out of a total of 25 marks?

0

$$\frac{18}{25} = \frac{18\times 4}{25\times 4} = \frac{72}{100} = 72\ \%$$

Example 3.

What percentage is 25 of 40?

0

$$\text{Fraction} = \frac{25}{40} = \frac{5}{8}$$

$$\frac{5}{8} = \frac{5}{8} \times 100\ \%$$

$$= \frac{500}{8}\ \%$$

$$= \frac{125}{2}\ \%$$

$$= 62\,\frac{1}{2}\ \%$$

So, 25 is $62\frac{1}{2}$ % of 40.

Example 4.

What percentage is 600 grams of 48 kg?

0

Fractions require the numerator and denominator to have the same sized parts. So, we will convert the kilograms to grams.

$$\text{Fraction} = \frac{600}{48000} = \frac{6}{480} = \frac{3}{240} = \frac{1}{80}$$

$$\frac{1}{80} = \frac{1}{80} \times 100\ \%$$

$$= \frac{10}{8}\ \%$$

$$= \frac{5}{4}\ \%$$

$$= 1\frac{1}{4}\ \%$$

So, 600g is $1\frac{1}{4}$ % of 48 kg.

Percentages also give us a convenient way to compare any number of *pairs of quantities* with different *bases* amounts. For example, suppose

- Ashley gave \$90 of her \$120 birthday windfall to charity,
- while Lin gave \$210 of her \$300 birthday windfall.

Who is the most generous? 🤔

A percentage is always *per 100*, and so percentages make it easier to compare two (or more) different situations like the Ashley and Lin case.

In this case, 90 is 75 % of 120 while 210 is 70 % of 300.
So, Ashley is more generous. Right? 🤔

Question 1

What percentage is:

a) 11 of 50

b) 3 of 20

c) 29 of 200

d) 12 of 60

e) 24 of 40

f) 18 of 24

g) 27 of 300

h) 12 of 96

i) 50 of 250

Question 2

What percentage is:

a) $12 of $36

b) $60 of $30

c) 45 m of 30 m

d) 320 g of 4 kg

e) 90 cm of 3 m

f) $2000 of $120 000

Question 3

An ice cream shop sold 800 scoops of ice cream on a Sunday afternoon. 520 of the scoops were vanilla. What percentage of the total number of scoops were vanilla?

Question 4

Lucy owns a book shop and in January she sold 250 books.
To increase sales, she paid for some radio advertising. In February she sold 450 books.
What percentage is the increase in sale of the January sales?

Question 5

Brenda claimed to have made 12 three-point goals from 30 attempts in practice last week.
Julie claimed she made 9 from 24 attempts. Who had the greater *conversion percentage*?

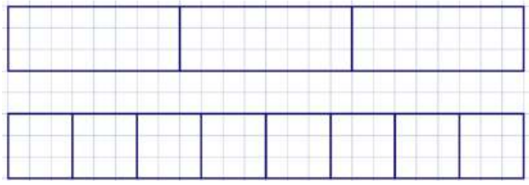
Question 6

A company makes go kart wheels. They tested two new methods of manufacturing wheels, Method A and Method B.
In one day of testing the following occurred.

- Method A produced 250 wheels, but 68 of them were defective and had to be discarded.
- Method B produced 380 wheels, but 114 of them were defective and had to be discarded.

Which method performed better?

20. Which fraction is greater/less than the other?



Which is greater, $\frac{2}{3}$ or $\frac{5}{8}$?

To figure this out, we could draw two identical rectangles and carefully choose their width to be a number that both 3 and 8 will divide into without remainder.
Such a number is called a common multiple of 3 and 8. Look up. ☺

Instead of using a diagram, we could approach the task numerically and find equivalent (equal) fractions for each fraction that have the same (common) denominator.
The two processes are the same.

Remember that the numerator of a fraction is like a count, a count of *how many unit fractions* are present.

So, $\frac{2}{3}$ is literally *2 one-thirds* and $\frac{5}{8}$ is literally *five one-eights* and so if we calculate equivalent (equal) fractions for each, *with the same denominator*, then we can compare them directly.

In this case we need to find a number that is *both* a multiple of 3 and a multiple of 8.
(What is a common multiple of 3 and 8?)
To do that we can ask, what number will both 3 and 8 divide into without remainder?
The *easiest* one to find, is the number with a factor of 3 *and a* factor 8, so we can multiply 3 by 8, which gives 24.

$$\frac{2}{3} = \frac{2\times8}{3\times8} = \frac{16}{24}$$

$$\frac{5}{8} = \frac{5\times3}{8\times3} = \frac{15}{24}$$

So we can now compare $\frac{16}{24}$ to $\frac{15}{24}$.
Since $16 > 15$ we can conclude that $\frac{2}{3} > \frac{5}{8}$.

There are other methods you could use to compare the size of two fractions.
Each method is the 'same', but 'different'.
You could express each fraction in decimal form and compare.
Or, express each fraction as a percentage and compare the percentages.

Example 1.

Which is greater, $\frac{5}{9}$ or $\frac{11}{18}$?

———— 0 ————

A common multiple of 9 and 18 is 18.
So, 18 can be a common denominator.

$$\frac{5}{9} = \frac{10}{18}$$

Compare $\frac{11}{18}$ to $\frac{10}{18}$.

Since 11 is greater than 10 we can conclude that $\frac{11}{18}$ is greater than $\frac{5}{9}$.

Example 2.

Which is greater, $\frac{7}{12}$ or $\frac{3}{5}$?

———— 0 ————

A common multiple of 12 and 5 is 60.
So, 60 can be a common denominator.

$$\frac{7}{12} = \frac{35}{60}$$

$$\frac{3}{5} = \frac{36}{60}$$

Since 36 is greater than 35 we can conclude that $\frac{3}{5}$ is greater than $\frac{7}{12}$.

Question 1

Find one common multiple for each of the following pairs of numbers.

a) 3 and 9
b) 4 and 20
c) 3 and 7
d) 9 and 12
e) 6 and 15
f) 30 and 36

Question 2

Determine which fraction of each pair is the largest.

a) $\frac{3}{5}, \frac{26}{45}$

b) $\frac{4}{7}, \frac{35}{56}$

c) $\frac{3}{5}, \frac{7}{11}$

d) $\frac{12}{7}, \frac{21}{12}$

e) $\frac{11}{6}, \frac{29}{15}$

f) $\frac{19}{30}, \frac{23}{36}$

Question 3

Accurately place the following fractions on the number line provided:

$$\frac{2}{3}, \frac{5}{12}, \frac{4}{10}$$

(Second number line supplied, just in case. ☺)

Question 4

Accurately place the following fractions on the number line provided:

$$\frac{2}{3}, \frac{3}{5}, \frac{5}{9}$$

Question 6

Draw your own number line and place the following fractions on it *reasonably* accurately.

$$\frac{10}{19}, \frac{20}{27}, \frac{27}{43}$$

21. Operations involving fractions – part 1

21.1 Dividing out prime factors

You more than likely know that 36 ÷ 9 = 4. ☺

Perhaps, however, you do not know the prime reason this is true.

$36 = 2 \times 2 \times 3 \times 3$.
$2 \times 2 \times 3 \times 3$ is the *prime factorisation* of 36.

$9 = 3 \times 3$.
3×3 is the *prime factorisation* of 9.

So,

$$36 \div 9 = \frac{36}{9} = \frac{2 \times 2 \times 3 \times 3}{3 \times 3} = 4$$

Prime factors that can be paired, one in the numerator and one in the denominator, can be *divided-out* since their division results in 1.

Note that this process is *equivalent* to the dividing the numerator and denominator of a fraction by the GCD, an idea you learned about to *simplify a fraction* in an earlier booklet. ☺

Example 1.

$$90 \div 10 = \frac{90}{10} = \frac{2 \times 3 \times 3 \times 5}{2 \times 5} = 9$$

Example 2.

$$42 \div 15 = \frac{42}{15} = \frac{2 \times 3 \times 7}{3 \times 5} = \frac{14}{5}$$

Question 1

Calculate each of the following by *dividing-out* prime factors.

a) $24 \div 8$

b) $36 \div 9$

c) $56 \div 8$

d) $96 \div 12$

e) $108 \div 18$

f) $216 \div 24$

Question 2

Write each of these fractions in simplest form by *dividing-out* prime factors.

a) $\frac{6}{8}$ c) $\frac{20}{25}$ e) $\frac{14}{35}$ g) $\frac{28}{56}$

b) $\frac{8}{12}$ d) $\frac{18}{30}$ f) $\frac{12}{42}$ h) $\frac{45}{30}$

21.2 The lowest common denominator (aka lowest common multiple)

Recall that to compare the size of two (or more) fractions we find equivalent fractions for each, that have the *same (common) denominator*.

For example, suppose we were required to compare $\frac{7}{9}$ to $\frac{11}{15}$.

What number can both 9 and 15 be 'multiplied up to'?

One way to answer this question is to find a number that both 9 and 15 will divide into, with zero remainder.

Such a number is called a common multiple of 9 and 15. There are lots of them!

Any one of the common multiples of 9 and 15 will do.

The easiest one to find is perhaps the product of the two denominators, i.e. $9 \times 15 = 135$.

But is 135 the smallest?

The smallest would be best, in this case, as it will make our work easier.

To find the smallest, we could write down the multiples of 9 and 15 and look for the ones that are common.

9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, 126, 135, 144, 153, 162, ...

15: 15, 30, 45, 60, 75, 90, 120, 135, 150, 165, ...

So, there was a smaller common multiple, 45.

45 is said to be the *lowest common multiple* (LCM) of 9 and 15.
We can also write, LCM(9,15) = 45.

There is an easier way (arguably) to find the LCM of a set of integers.

The LCM of 9 and 15 is: the number, *N*, with *the minimum* number of 9 *and* 15's prime factors so that *both* 9 *and* 15 *will divide into N with zero remainder.*

$9 = 3 \times 3$
$15 = 3 \times 5$

For 9 to divide into the LCM, the LCM will require exactly *two* factors of 3.
For 15 to divide into the LCM, the LCM will require exactly *one* factor of 3.
Therefore, the LCM, to be the lowest, requires *only* **two** factors of 3 (**not three**).

Also, for 15 to divide into the LCM, the LCM will require exactly one factor of 5.

So, LCM(9,15) = $3 \times 3 \times 5 = 45$.

In some cases, the LCM of two integers is the product of the two integers.
Can you think of what would be true about a pair of numbers for this to be the case?

Example 1.

What is the LCM of 6 and 8?

0

$6 = 2 \times 3$
$8 = 2 \times 2 \times 2$

So, three factors of 2 are needed **(not four)** and one factor of 3.

Therefore, LCM(6,8) $= 2 \times 2 \times 2 \times 3 = 24$.

Example 2.

What is the LCM of 15 and 18?

0

$15 = 3 \times 5$
$18 = 2 \times 3 \times 3$

So, one factor of 2, two factors of 3 **(not three)** and one factor of 5 are needed.

Therefore, LCM(15,18) $= 2 \times 3 \times 3 \times 5 = 90$.

Example 3.

What is the LCM of 24 and 90?

0

$24 = 2 \times 2 \times 2 \times 3$
$90 = 2 \times 3 \times 3 \times 5$

So, three factors of 2 (not four), two factors of 3 (not three) and one factor of 5 are needed.

Therefore, LCM(24,30) $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$.

Example 4.

What is the LCM of 30, 42 and 98?

0

$30 = 2 \times 3 \times 5$
$42 = 2 \times 3 \times 7$
$98 = 2 \times 7 \times 7$

So, one factor of 2, one factor of 3, one factor of 5 and two factors of 7.

Therefore, LCM(15,18) $= 2 \times 3 \times 5 \times 7 \times 7 = 1470$.

Question 1

Find the LCM of the following pairs of numbers.

a) 2 and 5
b) 3 and 10
c) 4 and 6
d) 6 and 10
e) 12 and 15
f) 12 and 21

Question 2

Find the LCM of:

a) 14 and 15
b) 16 and 22
c) 24 and 30
d) 2, 18 and 45
e) 6, 8 and 10
f) 3, 7 and 15

21.3 Addition involving fractions

You learned in an earlier section that the numerator of a fraction can be thought about as a *count*.

For example, the fraction $\frac{\mathbf{3}}{5}$ can be thought about as ***three*** $\frac{1}{5}$ s.

As such:

$$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{3}{5}$$

Similarly:

$$\frac{5}{7}+\frac{4}{7}=\frac{9}{7}=1\frac{2}{7}$$

If fractions have the *same* denominators, then we can add them by adding the numerators (the counts).

But things change if the denominators are not the same.

$\frac{5}{6}+\frac{1}{8}$ is **not** equal to 6 'any things', if we want the denominator to be a whole number.

When adding fractions with *different* denominators we first need to find an *equivalent* (equal) fractions for each with the *same* (common) denominator.

It is best to go for the *lowest common denominator* of the denominators involved, which is the LCM of the denominators. ☺

So, to calculate $\frac{5}{6}+\frac{1}{8}$:

- first find the LCM of 6 and 8, which is 24.
- then find the equivalent (equal) fraction for each with a denominator of 24.

$$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24} \text{ and } \frac{1}{8}=\frac{1\times3}{8\times3}=\frac{3}{24}$$

- then add the numerators (counts) of the two fractions with the same denominator.

$$\frac{20}{24}+\frac{3}{24}=\frac{23}{24}$$

In the examples that follow, you will see a nice way to set out fraction additions.

Example 1.

Calculate $\frac{1}{4}+\frac{5}{6}$

Give the result in simplest form.

$4 = 2\times2$
$6 = 2\times3$
$\therefore$ LCM(4,6) $= 2\times2\times3 = 12$

$\frac{1}{4}+\frac{5}{6}$

$= \frac{1\times3}{4\times3}+\frac{5\times2}{6\times2}$

$= \frac{3}{12}+\frac{10}{12}$

$= \frac{13}{12}$ (Check the numerator and denominator do not share any factors.)

$= 1\frac{1}{12}$

Example 2.

Calculate $\frac{3}{8}+\frac{7}{12}$

Give the result in simplest form.

$8 = 2\times2\times2$
$12 = 2\times2\times3$
$\therefore$ LCM(8,12) $= 2\times2\times2\times3 = 24$

$\frac{3}{8}+\frac{7}{12}$

$= \frac{3\times3}{8\times3}+\frac{7\times2}{12\times2}$

$= \frac{9}{24}+\frac{14}{24}$

$= \frac{23}{24}$ (Check the numerator and denominator do not share any factors.)

Example 3.

Calculate $\frac{2}{9}+\frac{4}{15}$

Give the result in simplest form.

$9 = 3\times3$
$15 = 3\times5$
$\therefore$ LCM(9,15) $= 3\times3\times5 = 45$

$\frac{2}{9}+\frac{4}{15}$

$= \frac{2\times5}{9\times5}+\frac{4\times3}{15\times3}$

$= \frac{10}{45}+\frac{12}{45}$

$= \frac{22}{45}$ (Check the numerator and denominator do not share any factors.)

Example 4.

Calculate $\frac{1}{4}+\frac{5}{9}$

Give the result in simplest form.

$4 = 2\times2$
$9 = 3\times3$
(Note, 4 and 9 are coprime so the LCM is the product of the two numbers.)
$\therefore$ LCM(4,9) $= 4\times9 = 36$

$\frac{1}{4}+\frac{5}{9}$

$= \frac{1\times9}{4\times9}+\frac{5\times4}{9\times4}$

$= \frac{9}{36}+\frac{20}{36}$

$= \frac{13}{36}$ (Check the numerator and denominator do not share any factors.)

Question 1

Calculate and give the result in simplest form.

a) $\frac{1}{6}+\frac{2}{3}$

b) $\frac{3}{8}+\frac{1}{4}$

c) $\frac{3}{5}+\frac{1}{3}$

d) $\frac{3}{8}+\frac{1}{10}$

e) $\frac{1}{8}+\frac{5}{6}$

f) $\frac{1}{12}+\frac{3}{10}$

Example 5.

Calculate $\frac{5}{15} + \frac{1}{9}$

Give the result in simplest form.

$\frac{5}{15}$ can be simplified to $\frac{1}{3}$ before starting and the LCM(3,9) = 3.

$$\frac{5}{15} + \frac{1}{9}$$

$$= \frac{1}{3} + \frac{1}{9}$$

$$= \frac{1\times 3}{9} + \frac{1}{9}$$

$$= \frac{3}{9} + \frac{1}{9}$$

$$= \frac{4}{9}$$ (Which is in simplest form.)

Example 6.

Calculate $\frac{1}{12} + \frac{3}{30}$

Give the result in simplest form.

$\frac{3}{30}$ can be simplified to $\frac{1}{10}$ before starting and the LCM(12,10) = 60.

$$\frac{1}{12} + \frac{3}{30}$$

$$= \frac{1}{12} + \frac{1}{10}$$

$$= \frac{1\times 5}{60} + \frac{1\times 6}{60}$$

$$= \frac{5}{60} + \frac{6}{60}$$

$$= \frac{11}{60}$$ (Which is in simplest form.)

Question 2

Calculate and give the result in simplest form.

a) $\frac{1}{4} + \frac{3}{6}$

b) $\frac{5}{25} + \frac{1}{10}$

c) $\frac{3}{21} + \frac{1}{14}$

d) $\frac{1}{8} + \frac{4}{24}$

e) $\frac{2}{10} + \frac{5}{40}$

f) $\frac{1}{6} + \frac{5}{20}$

Example 7.

Calculate $\frac{1}{4}+\frac{1}{12}$

Give the result in simplest form.

0

LCM(4,12) = 12.

$\frac{1}{4}+\frac{1}{12}$

$=\frac{1\times3}{12}+\frac{1}{12}$

$=\frac{3}{12}+\frac{1}{12}$

$=\frac{4}{12}$ (Which is NOT in simplest form.)

$=\frac{1}{3}$ (Which is in simplest form.)

Example 8.

Calculate $\frac{1}{6}+\frac{1}{14}$

Give the result in simplest form.

0

LCM(6,14) = 42.

$\frac{1}{6}+\frac{1}{14}$

$=\frac{1\times7}{42}+\frac{1\times3}{42}$

$=\frac{7}{42}+\frac{3}{42}$

$=\frac{10}{42}$ (Which is NOT in simplest form.)

$=\frac{5}{21}$ (Which is in simplest form.)

Question 3

Calculate and give the result in simplest form.

a) $\frac{1}{3}+\frac{1}{6}$

b) $\frac{3}{4}+\frac{1}{12}$

c) $\frac{11}{12}+\frac{1}{3}$

d) $\frac{5}{6}+\frac{2}{3}$

e) $\frac{5}{6}+\frac{5}{12}$

f) $\frac{1}{6}+\frac{7}{10}$

Question 4

Calculate and give the result in simplest form.

a) $\frac{8}{14}+\frac{1}{5}$

b) $\frac{4}{7}+\frac{2}{5}$

c) $\frac{4}{12}+\frac{1}{6}$

d) $\frac{4}{7}+\frac{5}{8}$

e) $\frac{7}{10}+\frac{3}{5}$

f) $\frac{9}{10}+\frac{10}{12}$

Question 5

Calculate and give the result in simplest form.

a) $\frac{1}{2}+\frac{1}{4}+\frac{1}{5}$

b) $\frac{2}{5}+\frac{2}{9}+\frac{1}{15}$

c) $\frac{1}{4}+\frac{5}{6}+\frac{3}{8}$

21.4 Subtraction involving fractions

The process of subtracting one fraction from another is the same as that for adding, but we subtract one count (numerator) from the other.

For example, 7-eighths subtract 5-eighths is equal to 2-eighths.
Or, in *fractional form*:

$$\frac{7}{8}-\frac{5}{8}$$

$$=\frac{2}{8}$$

$$=\frac{1}{4}$$

As for addition, if the denominators are different, we need to create an equivalent (equal) fraction for each, with a common denominator. Lowest common denominator is best.

Example 1.

Calculate $\frac{5}{6}-\frac{1}{4}$

Give the result in simplest form.

0

LCM(4,6) = 12.

$$\frac{5}{6}-\frac{1}{4}$$

$$=\frac{5\times 2}{12}-\frac{1\times 3}{12}$$

$$=\frac{10}{12}-\frac{3}{12}$$

$=\frac{7}{12}$ (Which is in simplest form.)

Example 2.

Calculate $\frac{7}{12}-\frac{3}{8}$

Give the result in simplest form.

0

LCM(8,12) = 24.

$$\frac{7}{12}-\frac{3}{8}$$

$$=\frac{7\times 2}{24}-\frac{3\times 3}{24}$$

$$=\frac{14}{42}-\frac{9}{42}$$

$=\frac{5}{24}$ (Which is in simplest form.)

Example 3.

Calculate $\frac{4}{12}-\frac{1}{9}$

Give the result in simplest form.

0

$\frac{4}{12}$ can be simplified to $\frac{1}{3}$ before starting.

LCM(3,9) = 9.

$$\frac{1}{3}-\frac{1}{9}$$

$$=\frac{1\times 3}{9}-\frac{1}{9}$$

$$=\frac{3}{9}-\frac{1}{9}$$

$=\frac{2}{9}$ (Which is in simplest form.)

Example 4.

Calculate $\frac{1}{4}-\frac{5}{9}$

Give the result in simplest form.

0

LCM(4,9) = 36.

$$\frac{1}{4}-\frac{5}{9}$$

$$=\frac{1\times 9}{36}-\frac{5\times 4}{36}$$

$$=\frac{9}{36}-\frac{20}{36}$$

$$=\frac{-11}{36}$$

$=-\frac{11}{36}$ (Which is in simplest form.)

Question 1

Calculate and give the result in simplest form.

a) $\frac{2}{3} - \frac{1}{6}$

c) $\frac{3}{5} - \frac{1}{3}$

e) $\frac{2}{10} - \frac{5}{40}$

b) $\frac{3}{8} - \frac{1}{4}$

d) $\frac{7}{21} - \frac{1}{9}$

f) $\frac{4}{20} - \frac{1}{6}$

Question 2

Calculate and give the result in simplest form.

a) $\frac{1}{8} - \frac{1}{5}$

b) $\frac{1}{6} - \frac{1}{5}$

c) $\frac{1}{9} - \frac{1}{3}$

Example 5.

Calculate $\frac{1}{4} - \frac{1}{12}$

Give the result in simplest form.

LCM(4,12) = 12.

$$\frac{1}{4} - \frac{1}{12}$$

$$= \frac{1 \times 3}{12} - \frac{1}{12}$$

$$= \frac{3}{12} - \frac{1}{12}$$

$= \frac{2}{12}$ (Which is in NOT simplest form.)

$= \frac{1}{6}$ (Which is in simplest form.)

Example 6.

Calculate $\frac{1}{6} - \frac{1}{14}$

Give the result in simplest form.

LCM(6, 14) = 42.

$$\frac{1}{6} - \frac{1}{14}$$

$$= \frac{1 \times 7}{42} - \frac{1 \times 3}{42}$$

$$= \frac{7}{42} - \frac{3}{42}$$

$= \frac{4}{42}$ (Which is NOT in simplest form.)

$= \frac{2}{21}$ (Which is in simplest form.)

Question 3

Calculate and give the result in simplest form.

a) $\frac{1}{3} - \frac{1}{6}$

b) $\frac{3}{4} - \frac{1}{12}$

c) $\frac{7}{12} - \frac{1}{3}$

d) $\frac{5}{6} - \frac{2}{3}$

e) $\frac{5}{6} - \frac{5}{12}$

f) $\frac{3}{10} - \frac{1}{6}$

Question 4

Calculate and give the result in simplest form.

a) $\frac{6}{14} - \frac{1}{5}$

b) $\frac{2}{5} - \frac{4}{7}$

c) $\frac{8}{12} - \frac{3}{18}$

d) $\frac{4}{7} - \frac{5}{8}$

e) $\frac{10}{45} - \frac{1}{10}$

f) $\frac{2}{12} - \frac{1}{10}$

Question 5

Calculate and give the result in simplest form.

a) $\frac{3}{4}-\frac{1}{8}+\frac{1}{5}$ b) $\frac{4}{5}-\frac{2}{9}-\frac{1}{15}$ c) $\frac{1}{4}+\frac{1}{6}-\frac{7}{8}$

21.5 Addition and subtraction with mixed numbers

We know that $3\frac{1}{7} = 3 + \frac{1}{7}$ and so if we had to add two mixed numbers, we could add the whole parts and then add the fraction parts separately.
For example,

$$5\frac{2}{7} + 3\frac{4}{7} = 5 + \frac{2}{7} + 3 + \frac{4}{7}$$

$$= 8 + \frac{6}{7}$$

$$= 8\frac{6}{7}$$

However, what happens if we subtract?

$$5\frac{2}{7} - 3\frac{4}{7} = 5 + \frac{2}{7} - 3 + \frac{4}{7}$$

= what do we do now? 😳 🤔

Calculating with mixed numbers can be messy!

One way to avoid any confusion when calculating with mixed numbers is to convert them to improper fractions before we start calculating.

Then you can apply all that you have learned in the previous sections.

A mixed number is in simplest form if its fraction part is in simplest form *and* less than 1.

Example 1.

Calculate $5\frac{2}{7} - 3\frac{4}{7}$

Give the result in simplest mixed number form.

0

$$5\frac{2}{7} - 3\frac{4}{7}$$

$$= \frac{37}{7} - \frac{25}{7}$$

$$= \frac{12}{7}$$

$$= 1\frac{5}{7}$$ (Which is in simplest form.)

Example 2.

Calculate $2 + 3\frac{4}{9}$

Give the result in simplest mixed number form.

0

$$2 + 3\frac{4}{9}$$

$$= 2 + 3 + \frac{4}{9}$$

$$= 5\frac{4}{9}$$ (Which is in simplest form.)

Example 3.

Calculate $7 - 3\frac{3}{5}$

Give the result in simplest mixed number form.

0

$$7 - 3\frac{3}{5}$$

$$= \frac{7}{1} - \frac{18}{5}$$

$$= \frac{7 \times 5}{5} - \frac{18}{5} \qquad (\text{LCM}(1,5) = 5)$$

$$= \frac{35}{5} - \frac{18}{5}$$

$$= \frac{17}{5}$$

$$= 3\frac{2}{5} \qquad (\text{Which is in simplest form.})$$

Note that you could have subtracted 3 (to reach 4) and then subtracted $\frac{3}{5}$ to reach the answer. ☺

Example 4.

Calculate $2\frac{4}{5} + 3\frac{1}{3}$

Give the result in simplest mixed number form.

0

$$1\frac{4}{5} + 1\frac{1}{3}$$

$$= \frac{9}{5} + \frac{4}{3}$$

$$= \frac{9 \times 3}{15} + \frac{4 \times 5}{15} \qquad (\text{LCM}(5,3) = 15)$$

$$= \frac{27}{15} + \frac{20}{15}$$

$$= \frac{47}{15}$$

$$= 3\frac{2}{15} \qquad (\text{Which is in simplest form.})$$

Question 1

Calculate and give the result in simplest form.

a) $5\frac{2}{7} + 3\frac{4}{7}$

b) $7\frac{5}{9} + 2\frac{4}{9}$

c) $7\frac{5}{9} + 2\frac{7}{9}$

d) $12 + 4\frac{3}{11}$

e) $8 - 2\frac{4}{7}$

f) $1\frac{1}{3} + \frac{2}{5}$

Question 2

Calculate and give the result in simplest form.

a) $2\frac{1}{2}+\frac{2}{3}$

b) $2\frac{1}{3}-\frac{3}{5}$

c) $2\frac{2}{3}-1\frac{3}{5}$

d) $1\frac{5}{7}+1\frac{2}{3}$

e) $4\frac{1}{2}-1\frac{3}{5}$

f) $4\frac{3}{8}+3\frac{10}{16}$

Example 5.

Calculate $4\frac{1}{10} - 2\frac{3}{4}$

Give the result in simplest mixed number form.

$$4\frac{1}{10} - 2\frac{3}{4}$$

$$= \frac{41}{10} - \frac{11}{4}$$

$$= \frac{41 \times 2}{20} - \frac{11 \times 5}{20} \qquad (\text{LCM}(4,10) = 20)$$

$$= \frac{82}{20} - \frac{55}{20}$$

$$= \frac{27}{20}$$

$$= 2\frac{7}{20} \qquad (\text{Which is in simplest form.})$$

Example 6.

Calculate $2\frac{2}{3} + 2\frac{1}{4}$

Give the result in simplest mixed number form.

$$2\frac{2}{3} + 2\frac{1}{4}$$

$$= \frac{8}{3} + \frac{9}{4}$$

$$= \frac{8 \times 4}{12} + \frac{9 \times 3}{12} \qquad (\text{LCM}(3,4) = 12)$$

$$= \frac{42}{15} + \frac{27}{15}$$

$$= \frac{69}{15}$$

$$= 4\frac{9}{15} \qquad (\text{Which is NOT in simplest form.})$$

$$= 4\frac{3}{5} \qquad (\text{Which is in simplest form.})$$

Question 3

Calculate and give the result in simplest form.

a) $1\frac{2}{5} + 3\frac{1}{2}$

b) $1\frac{1}{4} + 1\frac{1}{6}$

c) $4\frac{1}{6} - 1\frac{1}{8}$

d) $3\frac{5}{6} - 2\frac{2}{3}$

e) $1\frac{5}{6} + 2\frac{5}{12}$

f) $3\frac{1}{6} - \frac{7}{10}$

Question 4

Is $8\frac{14}{15} + 1\frac{1}{16}$ less than or greater than 10?

21.6 Miscellaneous questions

Question 1

Kim walked $\frac{7}{10}$ km while Jim walked $\frac{3}{5}$ km. How much further did Kim walk than Jim?

Question 2

Ashley used $\frac{2}{3}$ of one cup of milk to bake some cookies and $\frac{5}{6}$ of one cup of milk to bake a pudding. How many cups of milk were used in total?

Question 3

A painter mixes $\frac{2}{5}$ of one cup of blue paint with $\frac{1}{4}$ of one cup of red paint. How many cups of purple paint are created?

Question 4

A carton of milk contains 4 cups of milk. Ashley used $\frac{2}{3}$ of one cup of milk to bake some biscuits and $\frac{1}{4}$ of one cup of milk to bake make a small pudding. How much milk is left over?

Question 5

Lena estimates she completed about $\frac{3}{10}$ of her science project on the Tuesday. Rita estimated she completed $\frac{3}{5}$ of the same project on Tuesday.

a) Who completed more of the project on Tuesday?
b) How much more was done by the person who completed the most?

Question 6

Roger's car uses $\frac{3}{8}$ of one tank of petrol to drive from A to B, if he drives sensibly.
It uses $\frac{4}{7}$ of one tank of petrol to drive from B to C, if he drives sensibly.

a) Can Roger drive sensibly from A to C on one full tank of fuel?
b) If your answer to part a) is yes, then how much petrol will Roger have left.
   If no, then how much more petrol would Roger need?

Question 7

Anthony was raised on a $2\frac{1}{2}$ acre block of land, with a pine tree border. $\frac{3}{4}$ of one acre was used for chickens, $\frac{1}{8}$ of one acre was used for greyhound training and $\frac{1}{3}$ of one acre was used for the house and garden. How much of the block was unused?

Question 8

Figure out a method to calculate $5\frac{2}{5} - 3\frac{3}{4}$ *without* converting each mixed number to an improper fraction (or using a calculator).

Question 9

Figure out the numbers that go in each of the blank squares of this addition table.
If the number is less than 1, write it as a fraction in simplest form. Otherwise, write it as an improper fraction and mixed number in simplest form.

$+$		$\frac{1}{4}$	$\frac{2}{15}$	
$\frac{1}{3}$				$1\frac{1}{9}$
$\frac{5}{6}$		$\frac{13}{12} = 1\frac{1}{12}$		
			$\frac{5}{6}$	
$\frac{5}{12}$	$\frac{5}{8}$			

Question 10

In a group of music lovers, $\frac{1}{3}$ like classical music but nothing else and $\frac{1}{5}$ like rock music but nothing else. $\frac{1}{4}$ like both classical music and rock music but nothing else.

a) What fraction of the group do not like classical or rock music?

b) If the group consisted of 240 music lovers, how many do not like rock music?

Question 11

If a, b, c, d, e and f are different digits, find the *smallest* value *and* the *largest* value of the fraction sum $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$.

Question 12

Calculate each of the following and write the answer in both fractional and decimal form.

You might like to enlist the help of a calculator for at least part of the work.

a) $\frac{1}{4}+\frac{1}{16}$

b) $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}$

c) $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}$

Question 13

In Question 12, you calculated three steps of the *never-ending sequence of additions* (aka an infinite series):

$$\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}+\ldots$$

Take your time and calculate, carefully, 4 more steps of the never-ending sequence of additions.
Write the result of each step in both fractional and decimal form.

Do you notice anything interesting about the results?

Question 14

Here is a different infinite series for you to explore:

$$3 + \frac{4}{2\times3\times4} - \frac{4}{4\times5\times6} + \frac{4}{6\times7\times8} - \frac{4}{8\times9\times10} + \dots$$

Take your time and calculate, carefully, 20 steps of this infinite series.
Write the result of each step in decimal form (and fractional form for as long as your calculator can cope).

Do you notice anything interesting about the results?