

Worked solutions to the

QCAA Mathematical Methods External Assessment 2023 Paper 2 – Technology-active

Using the fx-CG50AU

These solutions present approaches to the mathematics in this assessment task. These solutions are not exhaustive, and other solutions methods are possible.

These solutions have been prepared by an experienced mathematics teacher, in-line with published information about QCAA assessment practices but in no way constitute an official set of solutions.



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External assessment 2023

Multiple choice question book

Mathematical Methods

Paper 2

General instruction

- Work in this book will not be marked.



Queensland
Government

QCAA

Queensland Curriculum
& Assessment Authority

Section 1

Instruction

- Respond to these questions in the question and response book.

QUESTION 1

If $f(x) = \sin(3x)$, determine the value of $f'\left(\frac{\pi}{8}\right)$.

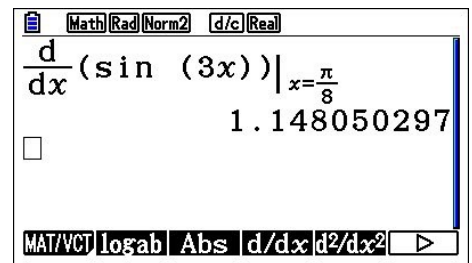
(A) 2.772

(B) 1.148

(C) 0.929

(D) 0.383

Access the derivative
command in Run-Matrix
via Math (F4) d/dx (F4)
Make sure your calculator
is in Radians



QUESTION 2

The probability of hitting a bullseye on a standard dartboard is 1 in 1250. What is the probability of hitting a bullseye exactly once in 10 attempts?

(A) $\binom{9}{1} \left(\frac{1}{1250}\right)^1 \times \left(\frac{1249}{1250}\right)^9$

Assuming a binomial
distribution $\text{Bin}(10, \frac{1}{1250})$

(B) $\binom{9}{1} \left(\frac{1}{1250}\right)^9 \times \left(\frac{1249}{1250}\right)^1$

Prob (one success in
10 attempts) = (C)

(C) $\binom{10}{1} \left(\frac{1}{1250}\right)^1 \times \left(\frac{1249}{1250}\right)^9$

(D) $\binom{10}{1} \left(\frac{1}{1250}\right)^9 \times \left(\frac{1249}{1250}\right)^1$

QUESTION 3

In a certain normal distribution curve, 95% of the area lies between the values 50.32 and 113.68. The mean of this distribution is 82.

Determine the standard deviation.

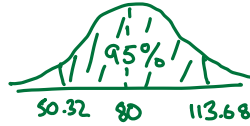
- (A) 16.16
 (B) 21.12
 (C) 31.68
 (D) 63.36

$$1.96 = \frac{113.68 - 82}{\sigma}$$

$$\therefore \sigma = \frac{113.68 - 82}{1.96}$$

$$= 16.16$$

Note: this area is centred about the mean i.e.



Note 2: the z-score associated with such an interval is $z = 1.96$, which can be found using InvN

	Rad	Norm2	d/c	Real
Inverse Normal				
Data	:	Variable		
Tail	:	Central		
Area	:	0.95		
σ	:	1		
μ	:	0		
Save Res:None				
None LIST				

	Rad	Norm2	d/c	Real
Inverse Normal				
x1 Inv	=	-1.959964		
x2 Inv	=	1.95996398		

QUESTION 4

The displacement (m) of a moving particle is given by $d = e^{0.5t} - 1$, where t is time (s).

The acceleration (ms^{-2}) of the particle when $t = 4$ is

- (A) 7.3891
 (B) 6.3891
 (C) 3.6945
 (D) 1.8473

Note: Acceleration is the second derivative of displacement

Note 2: Access the 2nd deriv. command in Run-Matrix via Math (F4) d^2/dx^2 (F

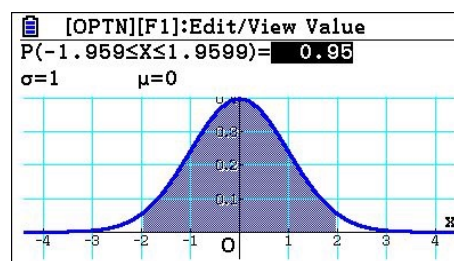
	Math	Rad	Norm2	d/c	Real
$\frac{d^2}{dx^2} (e^{0.5x} - 1) \Big _{x=4}$					
1.847264025					
MAT/VCT Logab Abs d/dx d ² /dx ²					

* Question 3 - alternative calculation of $z = 1.96$ using Distribution app.

Choose any lower and up - Tail central -

Arrow right and enter $P = 0.95$

	Rad	d/c
Normal		
Tail	:	Δ ($\leq X \leq$)
Lower	:	0
Upper	:	1
σ	:	1
μ	:	0
Execute		
EXE		



QUESTION 5

Solve $\ln(x) + \ln(3.70) = \ln(9.25)$ for x .

(A) 0.92

(B) 1.71

(C) 2.50

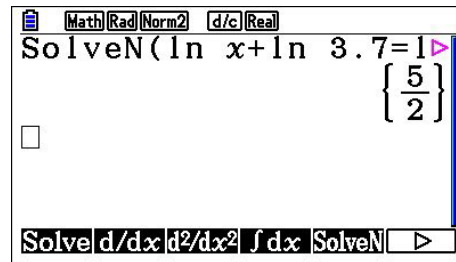
(D) 5.55

Method 1:

$$\ln(3.7x) = \ln(9.25)$$

$$\therefore 3.7x = 9.25$$

$$x = \frac{9.25}{3.7} = 2.5$$

Method 2:

QUESTION 6

$$\int_a^{5a} \frac{1}{x+a} dx, a \neq 0 \text{ is } = \left[\ln|x+a| \right]_a^{5a}$$

(A) 1.7918

(B) 1.6094

(C) 1.3863

(D) 1.0986

$$= \ln|6a| - \ln|2a|$$

$$= \ln\left(\frac{6a}{2a}\right)$$

$$= \ln 3$$

$$\approx 1.0986$$

QUESTION 7

The distribution of a certain sample proportion has a mean of 0.70 and a standard deviation of 0.02. Determine the sample size.

(A) 525

(B) 750

(C) 1750

(D) 2500

$$\text{mean of } \hat{p} = p = 0.7$$

$$\text{st. dev. of } \hat{p} = \sqrt{\frac{p(1-p)}{n}} = 0.02$$

$$\therefore \frac{0.7 \times 0.3}{n} = 0.02^2$$

$$\therefore n = \frac{0.7 \times 0.3}{0.02^2} = 525$$

Assuming that the sample is drawn from a population that can be modelled by a binomial distribution.

QUESTION 8

The number of koalas in a conservation park is modelled by $N = 15 \ln(7t + 1)$, $t \geq 1$, where t represents the time (years) since the park opened. There were 20 koalas in the park when it opened.

Determine the approximate rate of change in the number of koalas when $t = 3$.

Not relevant to rate of change

(A) 46

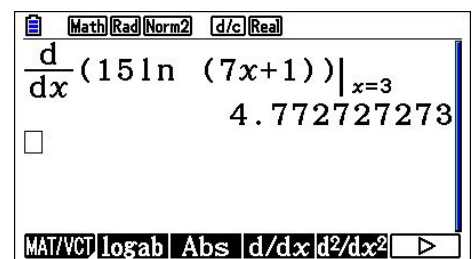
(B) 26

(C) 25

(D) 5

$\therefore$ Derivative value needed

Access the derivative command in Run-Matrix via Math (F4) d/dx (F4)



QUESTION 9

If $f(x) = e^{3x}(x+1)^2$ and $f'(x) = ae^{3x}(x+1)$, determine the expression for a .

- (A) $3x+5$ $f'(x) = 3e^{3x}(x+1)^2 + e^{3x} \times 2(x+1) \times 1$
 (B) $3x+3$ $= e^{3x}(x+1)[3(x+1) + 2]$
 (C) $5x+5$ $= (3x+5)e^{3x}(x+1)$
 (D) $5x+3$

QUESTION 10

A student is trying to determine which subject they performed best in compared to other students. Results from recent tests in four subjects (A to D) are shown. Assume student results in each subject are normally distributed.

In which subject did the student perform best compared to other students?

	Class mean	Class standard deviation	Student's result
(A)	62	22	77
(B)	55	25	74
(C)	61	15	70
(D)	73	20	82

Math	Rad	Norm2	d/c	Real
0.6818181818				
(74-55)÷25				
0.76				
(70-61)÷15				
0.6				
(82-73)÷20				
0.45				
JUMP DELETE ▶ MAT/VCT MATH				

Calculate the z-score of each result x using

$$z = \frac{x - \mu}{\sigma}$$

(as shown above)

Section 1

Instructions

- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil to fill in the A, B, C or D answer bubble completely.
- Choose the best answer for Questions 1–10.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	☒	☐	☐	☐

	A	B	C	D
1.	☐	☒	☐	☐
2.	☐	☐	☒	☐
3.	☒	☐	☐	☐
4.	☐	☐	☐	☒
5.	☐	☐	☒	☐
6.	☐	☐	☐	☒
7.	☒	☐	☐	☐
8.	☐	☐	☐	☒
9.	☒	☐	☐	☐
10.	☐	☒	☐	☐

Ensure you have filled an answer bubble for each question.

Do not write outside this box.

LUI

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School code

--	--	--	--

School name

--

Given name/s

--

Family name

--

Attach your
barcode ID label here

Book

--

 of

--

 books used

External assessment 2023

Question and response book

Mathematical Methods

Paper 2 – Technology-active

Time allowed

- Perusal time — 5 minutes
- Working time — 90 minutes

General instructions

- Answer all questions in this question and response book.
- QCAA-approved calculator **permitted**.
- QCAA formula book provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (45 marks)

- 9 short response questions



QUESTION 11 (4 marks)

A researcher found that 17 out of 50 randomly selected people had used public transport in the past week.

- a) Determine the sample proportion of people who had used public transport in the past week.

[1 mark]

$$\hat{p} = \frac{17}{50} = 0.34$$

- b) Determine an approximate 95% confidence interval for the proportion of people who had used public transport in the past week.

[2 marks]

$$0.34 - 1.96 \times \sqrt{\frac{0.34 \times 0.66}{50}} \leq p \leq 0.34 + 1.96 \times \sqrt{\frac{0.34 \times 0.66}{50}}$$
$$\therefore 0.2087 \leq p \leq 0.4713$$

- identify values required for CI
- calculate interval

Answer without working can be obtained using STAT-INT but marks scheme indicates working may be needed.

	Rad	Norm2	d/c	Real
1-Prop Z Interval				
Lower	=0.20869704			
Upper	=0.47130296			
$\hat{p}$	=0.34			
n	=50			

- c) Someone claims that: 50% of people use public transport each week.

Use your answer from Question 11b) to explain whether the data can or cannot support this claim.

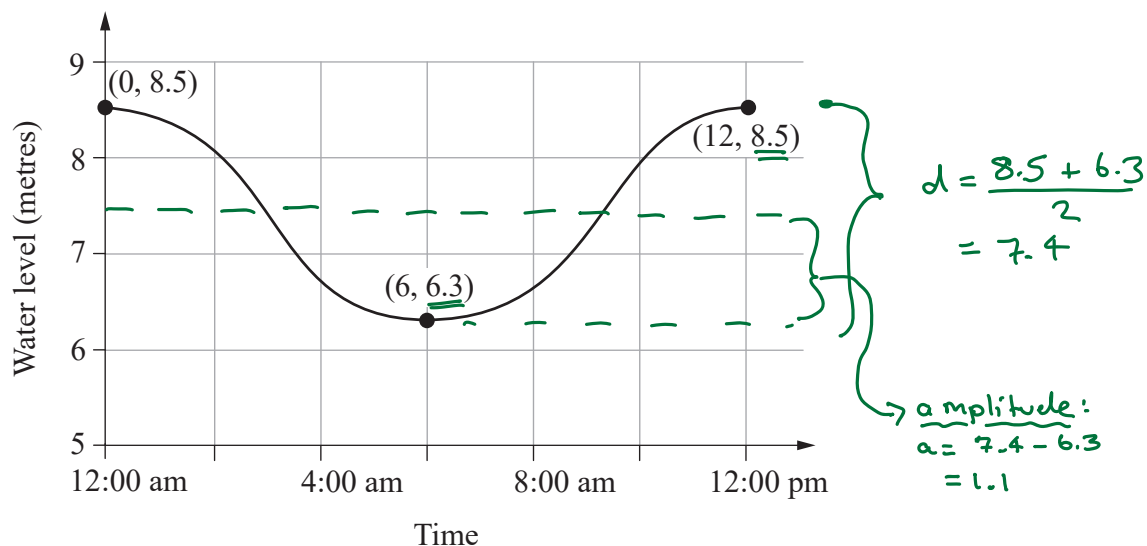
[1 mark]

Based on this data, we can be 95% confident that p , the true proportion of people who use public transport is between 21% and 47%. A claim of $p = 50\%$ lies outside this interval and so is not supported.

- evaluate claim using the above CI

QUESTION 12 (4 marks)

The graph shows the water level under a bridge over a 12-hour period.



- a) Determine the equation of the cosine function that models the water level as a function of time after 12:00 am.

[1 mark]

$$W(t) = 1.1 \cos\left(\frac{\pi}{6}t\right) + 7.4$$

Handwritten notes:

- $7.4 - 6.3$ (points to 1.1)
- $\frac{8.5 + 6.3}{2}$ (points to 7.4)
- $b = \frac{2\pi}{P} = \frac{2\pi}{12}$ (points to $\frac{\pi}{6}$)

- b) How long in the 12-hour period shown is the rate of change of water level more than 0.55 metres per hour?

[3 marks]

$$W(t) = -1.1 \sin\left(\frac{\pi}{6}t\right) + \frac{\pi}{6}$$

• Find $w'(t)$

$$w'(t) = 0.55 \text{ when}$$

• Solve $w'(t) = 0.55$

$$t = 8.4244 \text{ and } t = 9.5756$$

• Find time period

$$\therefore \text{time period} = 9.5756 - 8.4244$$

$$= 1.1512 \text{ hrs}$$

$$= 69 \text{ mins}$$

between the two solutions in appropriate units

Math Rad Norm2 d/c Real

SolveN $\left(-1.1 \sin\left(\frac{\pi}{6}x\right)\right) > 0.55$

{ 8.424418803, 9.57558 }

Solve d/dx d²/dx² ∫ dx SolveN

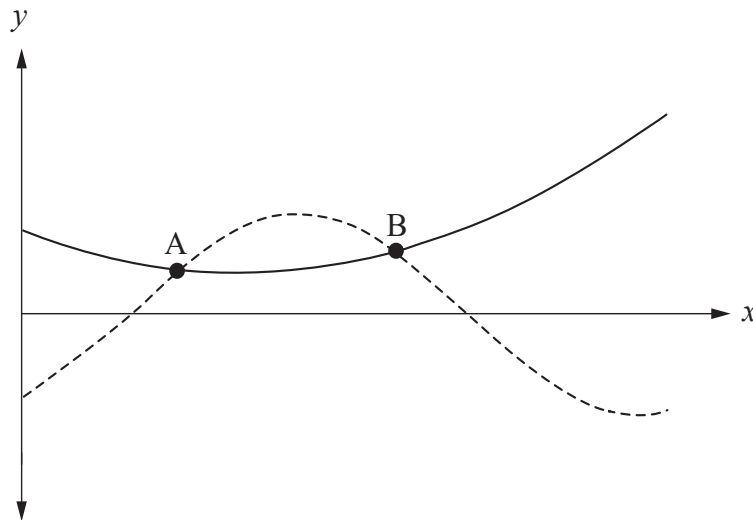
SolveN can solve

$w'(t) = 0.55$ without entering $w'(t) = -1.1 \sin \dots$ however marks scheme suggests working is required, so $w'(t)$ was found.

Do not write outside this box.

QUESTION 13 (4 marks)

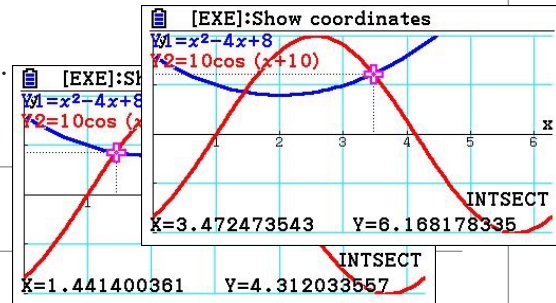
The curved lines represent graphs of the equations $y = x^2 - 4x + 8$ and $y = 10\cos(x+10)$.



- a) Determine the coordinates of the points of intersection A and B.

$A(1.44140, 4.31203)$
 $B(3.47247, 6.16818)$

• Use G-Solve - Intersect to find A & B



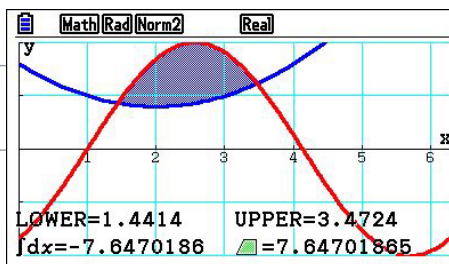
- b) State an integral expression representing the area enclosed by the two graphs. [2 marks]

$$\int_{1.44140}^{3.47247} 10\cos(x+10) - (x^2 - 4x + 8) dx$$

• Use x-values of A & B to write down integral
 (NB: watch order of subtraction)

- c) Determine the area enclosed by the two graphs. [1 mark]

7.64702 units^2

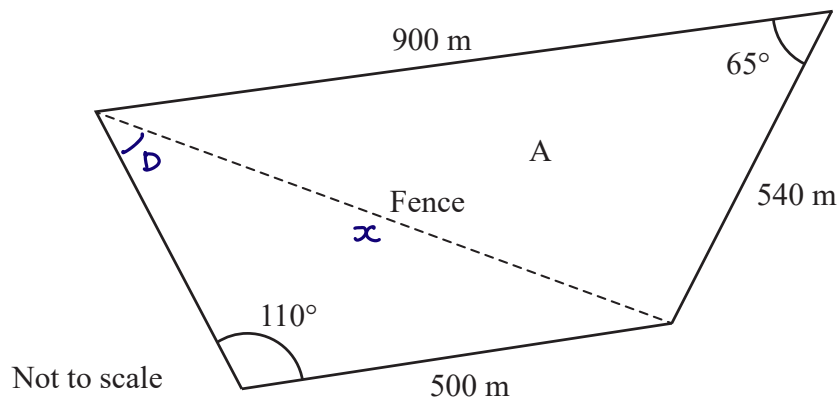


• Evaluate the integral / Find area
 (Graph app used here via G-Solve - Solve - Intersect)

Do not write outside this box.

QUESTION 14 (7 marks)

A fence divides a paddock into two triangular sections as shown.



- a) Determine the length of the fence.

[1 mark]

$$L_f = \sqrt{900^2 + 540^2 - 2 \times 900 \times 540 \times \cos 65}$$

$$= 831.1528 \text{ m}$$

• use Cosine Rule to find fence length

Math Deg Norm2 d/c Real

$\sqrt{900^2 + 540^2 - 2 \times 900 \times 540 \times \cos 65}$

831.1528437

☐

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- b) Calculate the area of triangular section A.

[1 mark]

$$\text{Area}_A = \frac{1}{2} \times 900 \times 540 \times \sin 65$$

$$= 220232.7922 \text{ m}^2$$

• Use Area of Triangle formula.

Math Deg Norm2 d/c Real

$\frac{1}{2} \times 900 \times 540 \times \sin 65$

220232.7922

☐

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Do not write outside this box.

c) Determine the total area of the paddock.

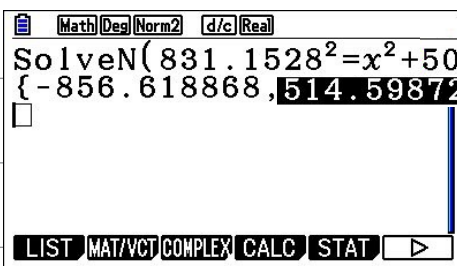
[5 marks]

Let x represent the missing length in the figure  • Use Cosine Rule to Set up equation involving missing side

$$831.1528^2 = x^2 + 500^2 - 2 \times x \times 500 \times \cos 110$$

$$\therefore x = 514.599 \text{ m} \quad (x > 0)$$

$$\begin{aligned} \text{Area}_B &= \frac{1}{2} \times 514.599 \times 500 \times \sin 110^\circ \\ &= 120891.2207 \text{ m}^2 \end{aligned}$$



• Solve equation using SolveN (NB: positive) sol. needed

$\therefore \text{Total Area} = 220232.7922 + 120891.2207 = 341124.0129 \text{ m}^2$ • Find the area of the Second triangle and Sum the two areas.

alternative solution using Sine Rule:

Let D represent the angle in the Second triangle opposite side of 500 m 

$$\therefore \frac{\sin D}{500} = \frac{\sin 110^\circ}{831.1528}$$

$$\therefore D = 34.4228^\circ$$

 third angle in the Second triangle $= 180 - (110 - 34.4228) = 35.5772^\circ$

$$\begin{aligned} \therefore \text{Area}_B &= \frac{1}{2} \times 500 \times 831.158 \times \sin 35.5772 \\ &= 120891.7987 \text{ m}^2 \end{aligned}$$

$$\therefore \text{Total Area} = 220232.7922 + 120891.7987 = 341124.5909 \text{ m}^2$$

 Marks Scheme and nature of question suggest that a clearly communicated solution showing key steps was needed.

QUESTION 15 (4 marks)

Determine the derivative of $f(x) = \ln x^2 + \ln(x-5)^3$. Express the derivative as a single fraction in its simplest and factorised form.

Using log laws on $f(x)$ first:

$$g(x) = \ln(x^2(x-5)^3)$$

$$\therefore g'(x) = \frac{2x(x-5)^3 + x^2 \times 3(x-5)^2 \times 1}{x^2(x-5)^3}$$

$$= \frac{\cancel{x}(x-5)^2 [2(x-5) + 3x]}{x^{\cancel{2}}(x-5)^{\cancel{3}}}$$

$$= \frac{5x-10}{x(x-5)} = \frac{5(x-2)}{x(x-5)}$$

• using $\ln A + \ln B = \ln AB$

• using $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$ ← Product Rule + Chain Rule

• take out common factor

• expand and factorise the numerator.

no log laws used:

$$g'(x) = \frac{\cancel{2}\cancel{x}}{x^{\cancel{2}}} + \frac{3(x-5)^2 \times \cancel{1}}{(x-5)^{\cancel{3}}}$$

$$= \frac{2(x-5) + 3x}{x(x-5)}$$

$$= \frac{5x-10}{x(x-5)} = \frac{5(x-2)}{x(x-5)}$$

• using $\frac{d}{dx}(\ln f(x)) = \frac{f'(x)}{f(x)}$ ← Chain Rule

(Simplifying fractions)

• making a common denominator

• expand and factorise the numerator

QUESTION 16 (6 marks)

A particle is moving in a straight line. The velocity (ms^{-1}) of the particle is given by

$$v(t) = \frac{20 \sin(2t)}{6 - 5 \cos(2t)}, t \geq 0, \text{ where } t \text{ is time (s) after moving from its initial position.}$$

The initial position of the particle is +6.0 m from the origin.

- a) Use calculus methods to determine an equation for the position of the particle from the origin at any time t .

[3 marks]

method of "u-substitution"

let $u = 6 - 5 \cos 2t \Rightarrow \frac{du}{dt} = 10 \sin 2t$ • define u and find $\frac{du}{dt}$

$$S(t) = \int \frac{20 \sin 2t}{6 - 5 \cos 2t} dt = \int 2 \times \frac{du}{dt} \times \frac{1}{u} dt$$

• Substitute u & $\frac{du}{dt}$ into integral

$$= 2 \int \frac{1}{u} du$$

• re-write integral in terms of u

$$= 2 \ln u + c = 2 \ln(6 - 5 \cos 2t) + c$$

• integrate w.r.t u then back-substitute $u = 6 - 5 \cos(t)$

$$S(0) = 2 \ln(6 - 5 \times \cos 0) + c = 6$$

• Use $S(0) = 6$ to find 'c'

$$\therefore c = 6 - 2 \ln 1 = 6$$

$$\therefore S(t) = 2 \ln(6 - 5 \cos 2t) + 6$$

$$S(t) = \int v(t) dt = 2 \int \frac{10 \sin 2t}{6 - 5 \cos 2t} dt$$

• manipulate so num. is deriv. of denom.

$$= 2 \ln(6 - 5 \cos 2t) + c$$

• use $\int \frac{f'(x)}{f(x)} dx = \ln f(x) + c$

$$S(0) = 2 \ln(6 - 5 \cos 0) + c = 6$$

$$\therefore c = 6$$

$$\therefore S(t) = 2 \ln(6 - 5 \cos 2t) + 6$$

⊗ A less structured version of the same thinking

- b) Determine the position of the particle relative to the origin when it first reaches maximum velocity.

[3 marks]

Particle first reaches max vel.

when $t = 0.292843$ seconds

option 1

$$S(0.292843) = 7.21227 \text{ m}$$

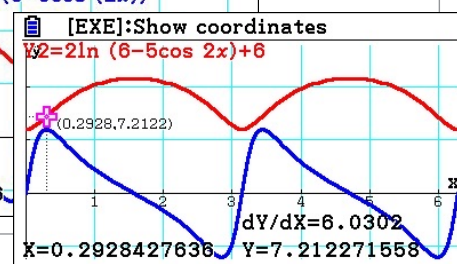
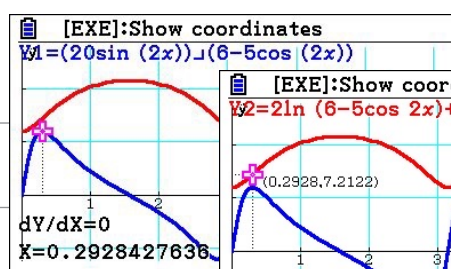
option 2 ⊗

$$\int_0^{0.292843} v(t) dt = 1.21227$$

but particle starts 6 m from 0

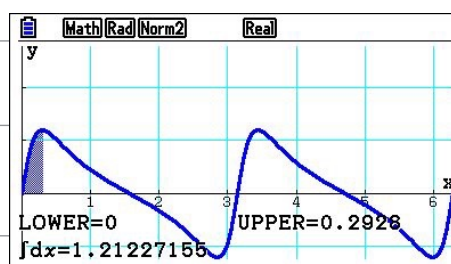
$$\therefore \text{pos. rel to 0} = 6 + 1.21227$$

$$= 7.21227 \text{ m}$$



• find max of $v(t)$ then

• (i) Sub t -value into $S(t)$



OR (2) Find area under $v(t)$ from 0 to t -value then add $S(0) = 6$

Do not write outside this box.

⊗ a good option if not confident about answer to part (a)

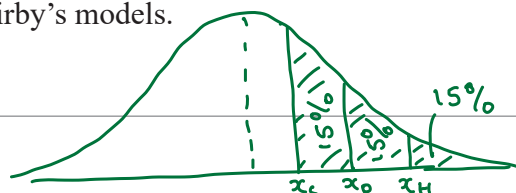
QUESTION 17 (5 marks)

Model bridges were constructed for a competition. The models that could support the heaviest loads before collapsing were given awards.

The load results of the competition were normally distributed, with a mean of 1.36 kg and a standard deviation of 0.12 kg.

Three award categories were used: honours for the top 15% of load results; distinction for the next 15%; and commendation for the next 15%.

The model bridge constructed by Finley only just missed out on a commendation. Kirby's model bridge only just qualified for honours. Determine the difference, to the nearest gram, between the loads supported by Finley and Kirby's models.



- Use InvNormal to find cut-offs of top 15% and 45%
- Find difference between cut-offs to nearest gram.

If $X \sim N(1.36, 0.12^2)$ then

$$Pr(X \geq x_H) = 0.15$$

$$\Rightarrow x_H = 1.48437 \text{ kg}$$

$$Pr(X \geq x_C) = 0.45 \leftarrow 0.15 + 0.15 + 0.15$$

$$\Rightarrow x_C = 1.37508 \text{ kg}$$

$\therefore$ maximum difference between Finley and Kirby's models is

$$1.48437 - 1.37508 = 0.10929$$

$$= 109 \text{ grams}$$

Rad	Norm2	d/c	Real
Inverse Normal			
Data	:	Variable	
Tail	:	Right	
Area	:	0.15	
σ	:	0.12	
μ	:	1.36	
Save	:		
None	:		
Inverse Normal			
xInv=1.48437201			

Rad	Norm2	d/c	Real
Inverse Normal			
Data	:	Variable	
Tail	:	Right	
Area	:	0.45	
σ	:	0.12	
μ	:	1.36	
Save	:		
None	:		
Inverse Normal			
xInv=1.37507936			

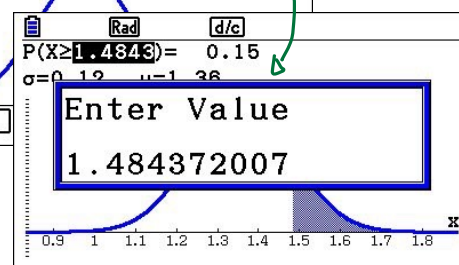
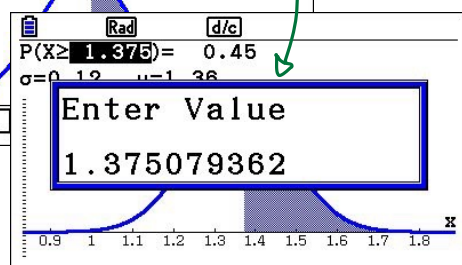
Alternative: Using Distribution app to find cut-offs

Rad	d/c
Normal	
Tail	: Δ (X $\geq$)
x	: 1
σ	: 0.12
μ	: 1.36
Execute	

- In Dist, choose right tail and enter any x
- Arrow right and enter Prob. use OPTN F1 to see full value.

[OPTN][F1]:Edit/View Value
P(X $\geq$ 1.375)= 0.45
 $\sigma=0.12$ $\mu=1.36$

[OPTN][F1]:Edit/View Value
P(X $\geq$ 1.4843)= 0.15
 $\sigma=0.12$ $\mu=1.36$



Do not write outside this box.



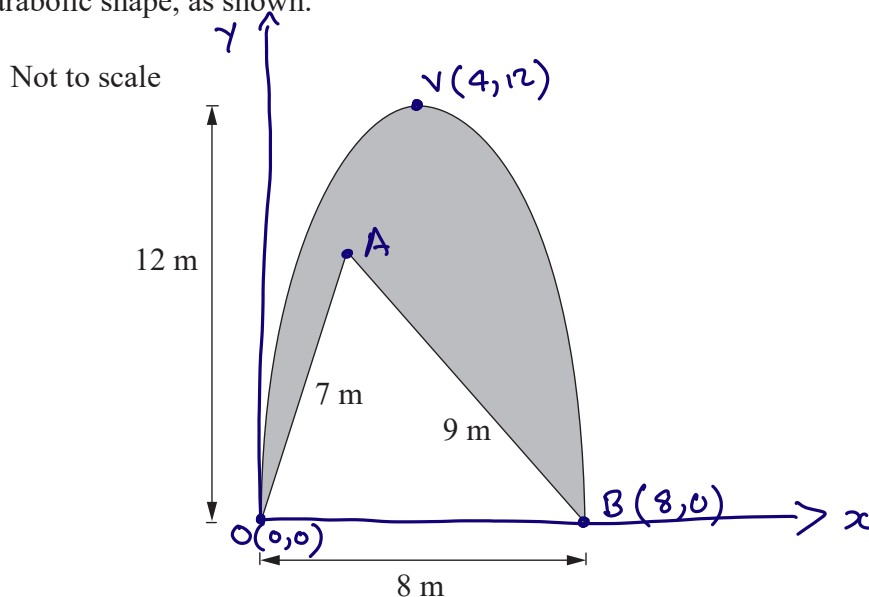
Do not write outside this box.



QUESTION 18 (5 marks)

marks scheme and nature of question
Suggests clear communication of key steps needed.

A company makes windows using glass that has a mass of 5.6 kg per square metre. A customer orders an unusual window in a partial parabolic shape, as shown.



Determine the mass of the window.

parabola with vertex $V(4, 12)$ has equation

$$y = a(x-4)^2 + 12, \text{ passing thru } (0, 0)$$

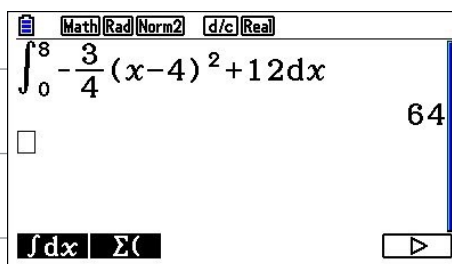
$$\therefore 0 = a(0-4)^2 + 12$$

$$\therefore a = -\frac{12}{16} = -\frac{3}{4}$$

- Use vertex given to find parabola in "turning point" form
- Sub in $(0, 0)$ or $(8, 0)$ to find "a"

- Use a definite integral

$$\begin{aligned} \therefore \text{Area under parabola} \\ &= \int_0^8 -\frac{3}{4}(x-4)^2 + 12 \, dx \\ &= 64 \, \text{m}^2 \end{aligned}$$



to find the area under the parabola

In triangle OAB

$$9^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos \hat{A}OB$$

$$\therefore \hat{A}OB = 73.3985^\circ$$

- Use cosine rule to find angle AOB

$$\begin{aligned} \therefore \text{Area of triangle OAB} \\ &= \frac{1}{2} \times 7 \times 8 \times \sin \hat{A}OB \\ &= 26.8328 \, \text{m}^2 \end{aligned}$$

- Find area of triangle AOB

Do not write outside this box.

∴ Area of glass

$$= 64 - 26.8328$$

$$= 37.1672 \text{ m}^2$$

∴ mass of glass

$$= 37.1672 \text{ m}^2 \times 5.6 \text{ kg/m}^2$$

$$= 208.136 \text{ kg}$$

- Find area of partial parabolic shape by subtraction of areas

- multiply m^2 by kg/m^2 to obtain mass in kg

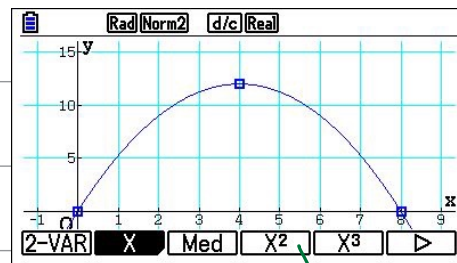
⊗ alternative method to find quadratic model:

- Enter pts $(0,0), (4,12), (8,0)$ into Statistics mode.

- Draw a Scatterplot

- Fit an " x^2 " regression model

NB: $y = -0.75x^2 + 6x$ is equiv. to
 $y = -\frac{3}{4}(x-4)^2 + 12$ as used above.



QuadReg
a = -0.75
b = 6
c = 0
 $r^2 = 1$
MSe =
 $y = ax^2 + bx + c$
COPY DRAW

QUESTION 19 (6 marks)

Over a suitable domain, a hill has a cross-sectional area given by $\int h(x) dx = \frac{a}{b} e^{bx} + c$, where:

- a , b and c are constants, $b \neq 0$
- $h(x)$ represents vertical distance (m), x represents horizontal distance (m).

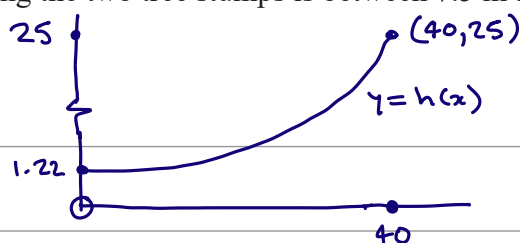
It is known that $h(0) = 1.22$ and $h(40) = 25$.

Where the gradient of the hill is 0.86 there is a tree stump. A second tree stump is located further up the hill. The difference in hill gradient between the two tree stumps is 0.44.

A surveyor predicts that the vertical distance separating the two tree stumps is between 7.5 m and 8.5 m. Evaluate the reasonableness of this prediction.

$$\int h(x) dx = \frac{a}{b} e^{bx} + c$$

$$\Rightarrow h(x) = \frac{d}{dx} \left(\frac{a}{b} e^{bx} + c \right) = a e^{bx}$$



$$(0, 1.22) \Rightarrow 1.22 = a \times e^0 \therefore a = 1.22$$

$$(40, 25) \Rightarrow 25 = 1.22 \times e^{40b}$$

* could solve this with 'SolveN'

$$\therefore e^{40b} = \frac{25}{1.22}$$

$$\therefore 40b = \ln\left(\frac{25}{1.22}\right)$$

$$\therefore b = \frac{1}{40} \ln\left(\frac{25}{1.22}\right) \approx 0.0755$$

$$h'(x) = 1.22 e^{0.0755x} \times 0.0755$$

$$h'(x_1) = 0.86 = 1.22 e^{0.0755x_1} \times 0.0755$$

$$x_1 = \ln\left(\frac{0.8}{1.22 \times 0.0755}\right) \approx 29.58837$$

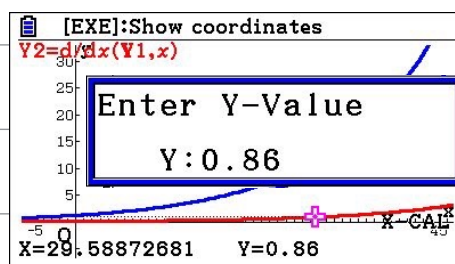
$$h(29.58837) = 11.3907$$

$\therefore$ vert. height of tree stump @ gradient of 0.86 is 11.3907 m

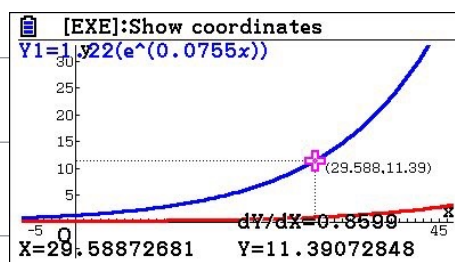
• Diff. to Find $h(x)$ in terms of a and b

• use points given to find a and b ⊕

• use first gradient to find x -value (use x-Calc or SolveN)



• Sub x_1 into $h(x)$ to find y -value (use Substitution or Trace)



Do not write outside this box.

$$h'(x_2) = 0.86 + 0.44 = 1.22 e^{0.0755x_2} \times 0.0755$$

$$x_2 = 35.0614$$

$$h(35.0614) = 17.2185$$

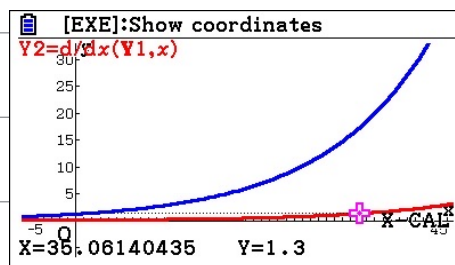
∴ vert. height of stump at gradient of 1.3 is 17.2185 m

∴ vertical distance between stump 1 and stump 2 is

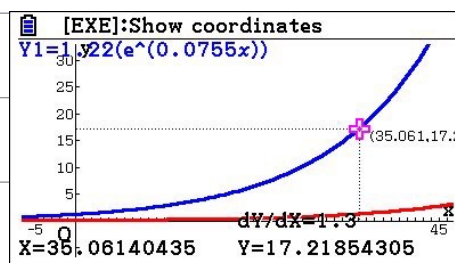
$$17.2185 - 11.3907 = 5.8278 \text{ m}$$

As the actual distance is not within the prediction of 7.5 to 8.5 m, the prediction is not reasonable (correct).

• Use gradient = 0.86 + 0.44 to find Second x-value



• Sub x_2 into $h(x)$ to find y-value



⊕ Alternative method to Find $h(x)$:

• Enter pts (0, 1.22) and (40, 25) into Statistics mode.

• Draw a Scatterplot

• Fit an " ae^{bx} " model

