SHRIRO | CASIO | EDUCATION

Worked solutions to the

QCAA Mathematical Methods External Assessment 2024

Paper 2 - Technology Active

Using the fx-CG50AU

Prepared by:
Casio Education Australia | edusupport@shriro.com.au

569001826

External assessment 2024

Multiple choice question book

Mathematical Methods

Paper 2 — Technology-active

General instruction

- Work in this book will not be marked.

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Section 1

Instruction

- Respond to these questions in the question and response book.

QUESTION 1

The probability of hitting a target in a particular binomial experiment is 0.72

Determine the mean of the number of hits if this experiment is repeated eight times.

(A) 1.61

(B) 2.24

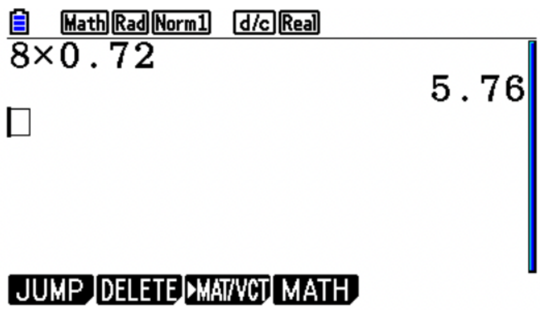
(C) 5.76

(D) 7.28

mean of binomial dist = np

$\mu = np$
$= 8 \times 0.72$
$= 5.76$

In the run mode:



QUESTION 2

Calculate the expected value of a continuous random variable X with the probability density function

$$p(x) = \begin{cases} \frac{1}{4}x^2, & 0 \le x \le \sqrt[3]{12} \\ 0, & \text{otherwise} \end{cases}$$

(A) 1.72

(B) 1.15

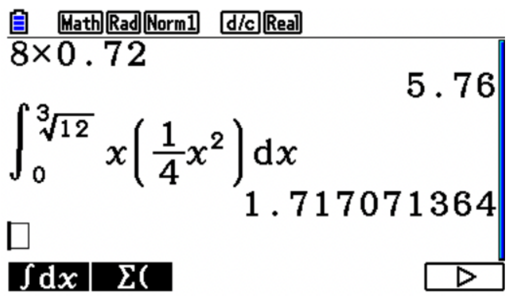
(C) 1.00

(D) 0.11

$$E[X] = \int_0^{\sqrt[3]{12}} x\left(\frac{1}{4}x^2\right) dx = 1.72$$

Given only 1 mark, do integration on calculator

To access the integration command on CG50:
Run mode
MATH F4
▷ F6
∫dx F1



QUESTION 3

The derivative of the function $f(x)$ is given by $f'(x) = \sin(2x)$. It is known that $f\left(\frac{\pi}{2}\right) = 4$.

Determine $f(x)$.

(A) $-\cos(2x)+3$

(B) $\cos(2x)+5$

(C) $-\frac{1}{2}\cos(2x)+3.5$

(D) $\frac{1}{2}\cos(2x)+4.5$

$f'(x) = \sin(2x)$

$f(x) = \int \sin(2x)\,dx$ $\{f(x) = \int f'(x)\}$

$= -\frac{1}{2}\cos(2x) + c$

Sub in $f(\pi/2) = 4$ to solve for c

$4 = -\frac{1}{2}\cos(2 \times \pi/2) + c$

$4 = -1/2(-1) + c$

$\therefore c = 3.5 \Rightarrow f(x) = -\frac{1}{2}\cos(2x) + 3.5$

QUESTION 4

Consider the Bernoulli distribution where the outcomes for rolling a six-sided die are a four and not rolling a four.

Determine the variance of the resulting Bernoulli distribution in this scenario.

(A) $0.02\dot{7}$

(B) $0.13\dot{8}$

(C) $0.1\dot{6}$

(D) $0.8\dot{3}$

Variance = $p(1-p)$

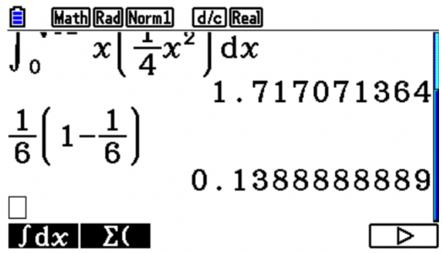
p = prob of 4 = $1/6$

$\therefore$ variance $= \frac{1}{6}\left(1-\frac{1}{6}\right)$

$= \frac{1}{6} \times \frac{5}{6}$

$= 5/36$

In the run mode:



QUESTION 5

The mass (g) of adult kookaburras in a certain region is normally distributed with a mean of 300 g and a standard deviation of 13 g. Select the correct statement about the mass of adult kookaburras.

(A) 34% are between 287 g and 313 g

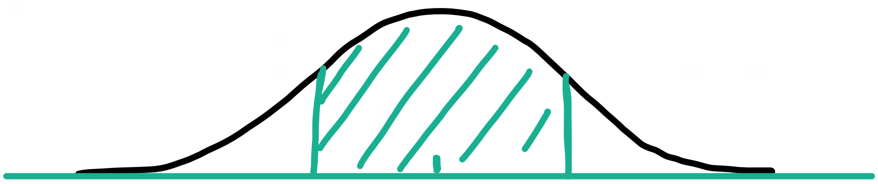
(B) 68% are between 274 g and 326 g

(C) 95% are between 261 g and 326 g

(D) 99.7% are between 261 g and 339 g

Let m = mass of adult kookaburra

$m \sim N(300, 13^2)$



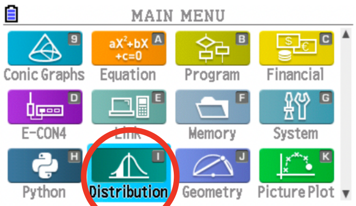
300

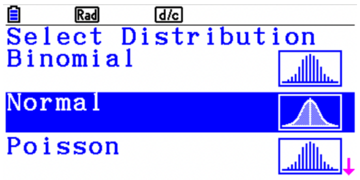
We can test each answer & see which is correct.

* See next page for calculator steps.

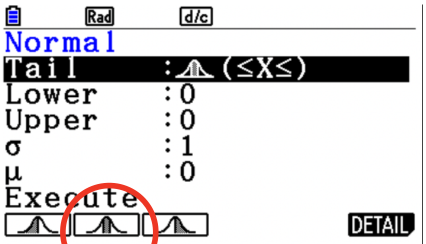
Method 1

Use distribution app & Select normal

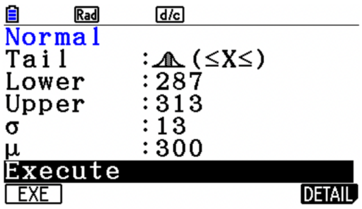


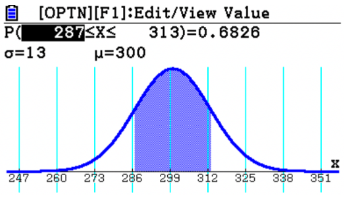


Use F2 to select the correct tail direction



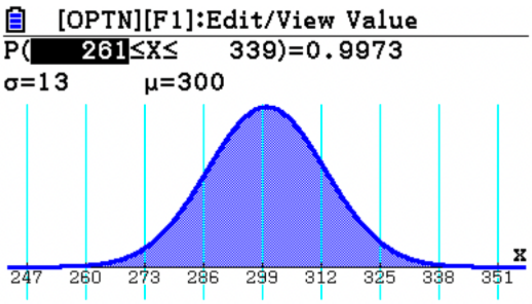
We can test each multiple choice answer by entering in the bounds of the mass





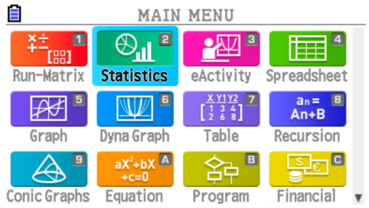
not this one!

Keep checking the multiple choice options Until you get the right one:

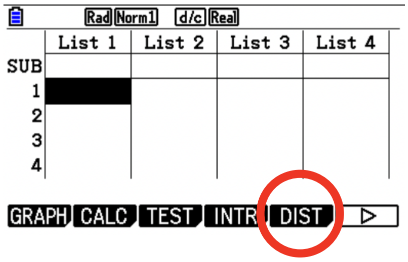


Method 2

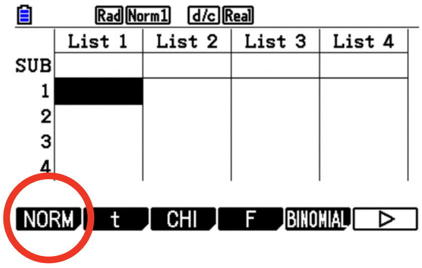
Use statistics app



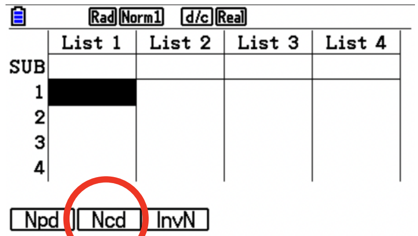
Dist (F5), Norm (F1), NCD (F2)



F5

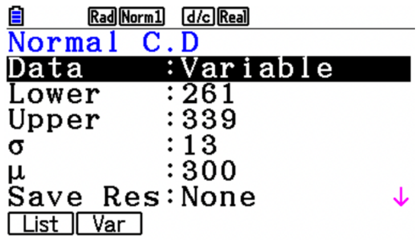


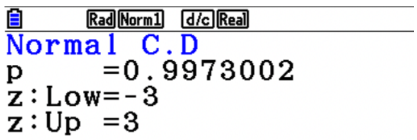
F1



F2

We can test each multiple choice option
Below we show the calculator input for the correct answer





Using either method, the correct answer is C

99.7% are between 261g & 339g.

Method 3

Using 68/95/99.7% rule can also be used here.

QUESTION 6

Determine the derivative of $y = 2x\cos(3x)$

(A) $2\cos(3x) - 6x\sin(3x)$

(B) $2\cos(3x) + 6x\sin(3x)$

(C) $-6\sin(3x)$

(D) $-2\sin(3x)$

$y = 2x\cos(3x)$

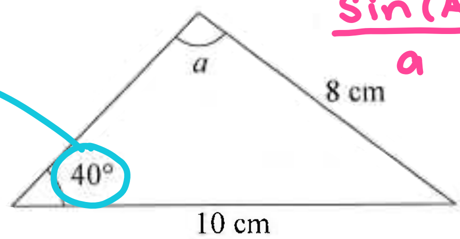
$\frac{dy}{dx} = 2\cos(3x) + 2x[-3\sin(3x)]$

$= 2\cos(3x) - 6x\sin(3x)$

QUESTION 7

Not to scale

Calc in degrees!!



Sine rule

$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$

$\frac{\sin(40)}{8} = \frac{\sin(a)}{10}$

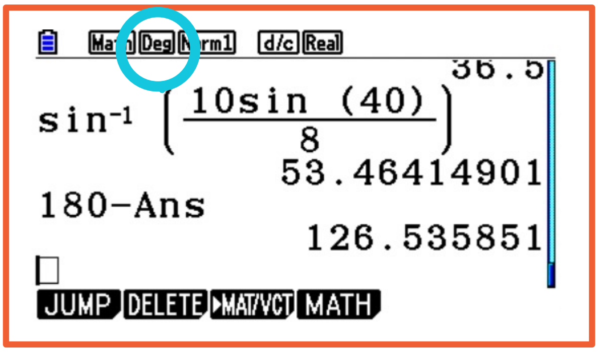
Identify the possible values for a in the triangle.

(A) 13.5° or 126.5°

(B) 53.5° or 126.5°

(C) 53.5° or 86.5°

(D) 13.5° or 86.5°



$\sin(a) = \frac{10\sin(40)}{8}$

$a = \sin^{-1}\left(\frac{10\sin(40)}{8}\right)$

$a = 53.5$

and $180 - a = 126.5$

QUESTION 8

Calculate the total enclosed area between the graph of $y = x^2 - x - 6$ and the x-axis from $x = 1$ to $x = 5$

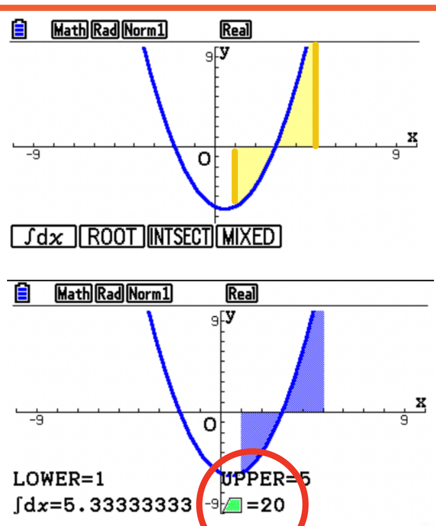
(A) 5.33

(B) 7.33

(C) 12.67

(D) 20.00

NOTE: area not integral



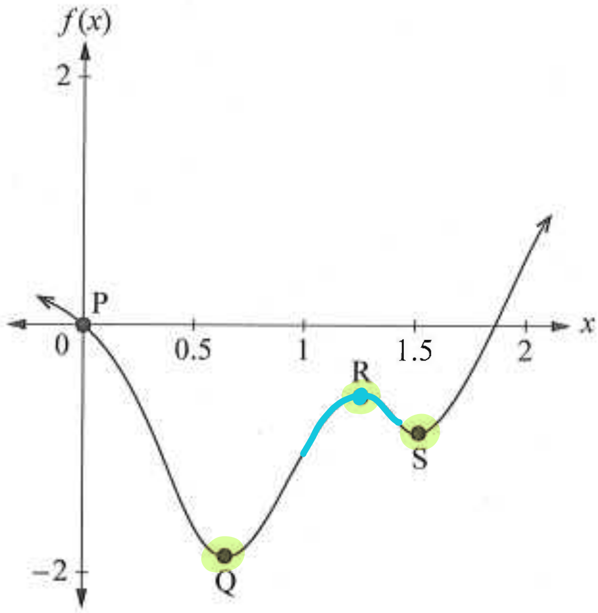
Plotting $y = x^2 - x - 6$, we in the graph mode, we can see the area (shaded yellow) is both above & below the x-axis. Thus, to find the area

GSolv ▷ F5 F6 } ∫dx F3 Mixed F4

Thus the area is 20.

QUESTION 9

It is known that $f'(x) = 0$ and $f''(x) < 0$ for one of the labelled points on the graph of $f(x)$.



Which point matches this description?

(A) P

(B) Q

(C) R

(D) S

Look for a point where $f'(x) = 0$
either : Q, R, S

$f''(x) < 0 \Rightarrow f(x)$ is concave down
i.e. $\cap$

$\therefore$ the point R matches the description.

QUESTION 10

The velocity (m s^{-1}) at time t (s) of an object is given by $v(t) = 0.4t^2 + 3t$ for $t \geq 0$.

The change in displacement (m) of the object from four to five seconds is

(A) 15.43

(B) 21.63

(C) 32.53

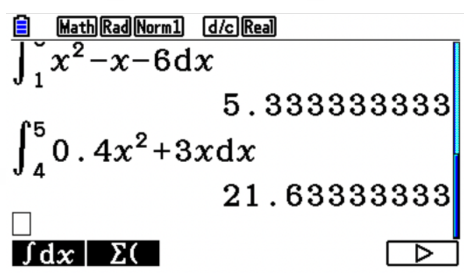
(D) 54.17

$s(t) = \int v(t)\,dt$

$s(t) = \int_4^5 0.4t^2 + 3t \; dt$

$= 21.63$

In the run mode,
MATH F4
▷ F6
$\int dx$ F1



Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this book.
  - On the additional pages, write the question number you are responding to.
  - Cancel any incorrect response by ruling a single diagonal line through your work.
  - Write the page number of your alternative/additional response, i.e. See page …
  - If you do not do this, your original response will be marked.
- This section has nine questions and is worth 45 marks.

QUESTION 11 (4 marks)

State the trapezoidal rule and use it with six strips to determine an approximate value of the definite integral for the curve of $f(x) = 4(x-3)^2$ from $x = 0$ to $x = 3$. Show all substitutions made into the rule.

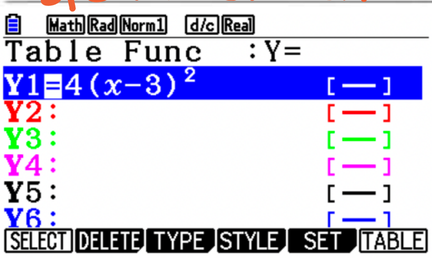
$A = \frac{1}{2} w \left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_n) \right]$ (1)

$w = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2}$ (1)

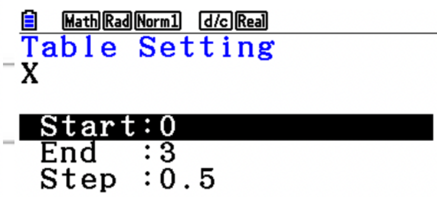
$A = \frac{1}{2} \times \frac{1}{2} \left[f(\tfrac{1}{2}) + 2f(1) + 2f(1\tfrac{1}{2}) + 2f(2) + 2f(2\tfrac{1}{2}) + f(3) \right]$

To evaluate the f^n, use the table app:

Type in function

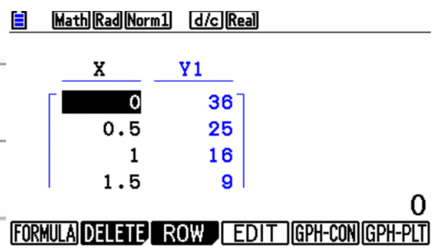


set values (F5)



click F6 to see table

use down arrow to see all values



$= \frac{1}{4} \left[36 + 2(25) + 2(16) + 2(9) + 2(4) + 2(1) + 0 \right]$ (1)

easy to evaluate in run mode.

$= 36.5 \text{ units}^2$ (1)

Do not write outside this box.

0053GZ0Q204

QUESTION 12 (5 marks)

The magnitude of an earthquake can be modelled by the logarithmic equation $M_A = \log_{10}\left(\frac{I_A}{I_0}\right)$, where M_A is the magnitude at a location A, I_A is the intensity of the earthquake at location A and I_0 is a constant.

An earthquake at location P had a magnitude of 5.2.

A different earthquake at location Q had a magnitude of 3.5.

a) Determine an equation involving logarithms that expresses the difference in magnitudes between the earthquakes at locations P and Q. *[1 mark]*

At P: $M_P = \log_{10}\left(\frac{I_P}{I_0}\right)$ At Q: $M_Q = \log_{10}\left(\frac{I_Q}{I_0}\right)$

$\therefore$ difference $= M_P - M_Q = \log_{10}\left(\frac{I_P}{I_0}\right) - \log_{10}\left(\frac{I_Q}{I_0}\right)$ (1)

b) How many times more intense was the earthquake at location P than the earthquake at location Q? *[4 marks]*

$M_P = 5.2$ & $M_Q = 3.5$

$\Rightarrow 5.2 - 3.5 = \log_{10}\left(\frac{I_P}{I_0} \div \frac{I_Q}{I_0}\right)$ (1)

$\log(A) - \log(B) = \log\left(\frac{A}{B}\right)$

$1.7 = \log_{10}\left(\frac{I_P}{I_Q}\right)$ (1)

$10^{1.7} = \frac{I_P}{I_Q}$ (1)

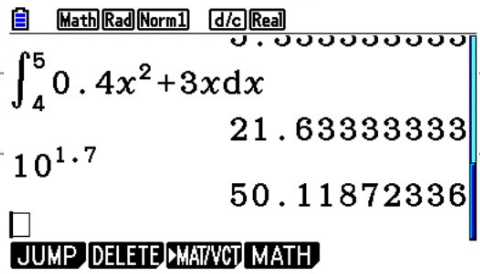
$10^{\log_{10}(x)} = x$

$I_P = 10^{1.7} I_Q$

$I_P = 50.12\, I_Q$ (1)

The intensity at P is 50.12 times that of Q.

In the run mode:



Do not write outside this box.

0053GZ0Q205

QUESTION 13 (8 marks)

The number of termites in a particular nest can be modelled by $N(t) = \dfrac{A}{2+e^{-t}}$, where A is a constant and t represents time (months) since the nest first became a visible mound above ground level.

It is estimated that when the mound first became visible, the population was 3×10^5 termites.

a) Determine the value of A. *[1 mark]*

when the mound first becomes visible $\Rightarrow t=0$

$$3\times10^5 = \frac{A}{2+e^0}$$

$\therefore A = 3\times10^5(3)$

$\therefore A = 900000$ (1)

In the run mode:

21.63333333
$10^{1.7}$
50.11872336
$3\times10^5\times3$
900000

JUMP DELETE MAT/VCT MATH

b) Determine the number of termites in the nest half a year after the mound became visible. *[2 marks]*

Given t is in months, need $t=6$ to represent 6 months. (1)

In the run mode:

$3\times10^5\times3$
900000
$\frac{900000}{2+e^{-6}}$
449442.9711

JUMP DELETE MAT/VCT MATH

$$N = \frac{900000}{2+e^{-6}} = 449442.97 \approx 449443 \text{ termites} \quad (1)$$

c) Determine the time in months after the mound became visible for the initial population to increase by 130 000 termites. Express the time as a decimal. *[2 marks]*

$N = 300000 + 130000 = 430000$ (1)

Want to find t s.th $N = 430000$

$$430000 = \frac{900000}{2+e^{-t}} \Rightarrow t = 2.375 \quad (1)$$

Since worth 2 marks, use solve N (OPTN CALC F4 SolveN F5)

$2+e^{-6}$
449442.9711
$\text{SolveN}\left(430000 = \frac{900000}{2+e^{-x}}\right)$
$\{2.374905755\}$

Solve d/dx d²/dx² ∫dx SolveN ▷

Do not write outside this box.

0053GZ0Q206

d) Develop a formula for the rate of change in the number of termites at any time after the mound became visible. Express your formula as a fraction. *[2 marks]*

rate of change => derivative

$$N = \frac{900000}{2+e^{-t}} = 900000(2+e^{-t})^{-1}$$

$$N'(t) = -900000(2+e^{-t})^{-2}(-e^{-t}) \quad (1)$$

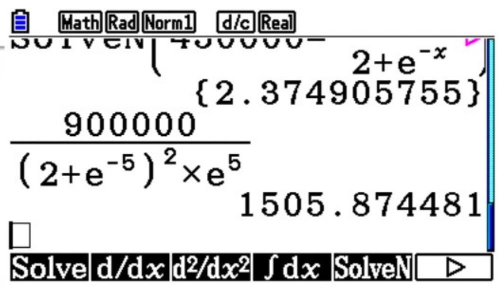
$$= \frac{900000}{(2+e^{-t})^2 e^{t}} \quad (1)$$

{make sure to express as a fraction}

e) Determine the rate of change in the number of termites five months after the mound became visible. *[1 mark]*

$$N'(5) = \frac{900000}{(2+e^{-5})^2 \times e^5} = 1505.87 \approx 1506 \quad (1)$$

In the run mode:



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QUESTION 14 (6 marks)

A football coach offered a 12-day intensive training clinic. During the clinic, the height that each player could kick a football was monitored.

One player's kick heights could be modelled by $H(t) = \log_{10}(10t+10)+5,\ 0 \le t \le 12$, where $H(t)$ is vertical height (m) and t is the time (days) spent in training.

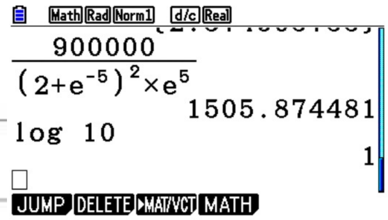
a) Determine the initial height that the player could kick the ball.

initial height $\Rightarrow t=0$

$H(0) = \log_{10}(10) + 5$

$= 1 + 5$

$= 6$ (1)



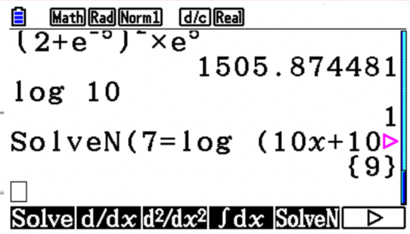
b) Determine the training time needed for the player to be able to kick the ball to a height of 7 m.

$7 = \log_{10}(10t+10)+5$

Solve using SolveN

$\therefore t = 9$ days (1)

In the run mode:
OPTN
Calc F4
SolveN F5



c) Determine the overall improvement in kick height achieved by completing the clinic. *[2 marks]*

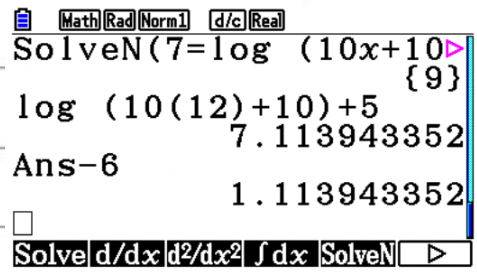
The clinic runs for 12 days

$H(12) = \log_{10}(10(12)+10)$

$= 7.1139$ (1)

$7.1139 - 6 = 1.1139$

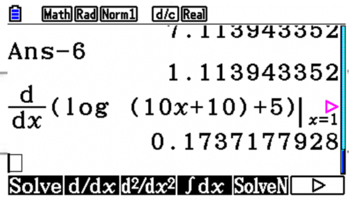
$\therefore$ An increase of 1.1139m in kick height. (1)



d) Determine the rate of change in kick height when $t = 1.5$ days. *[1 mark]*

Use GC as 1 mark

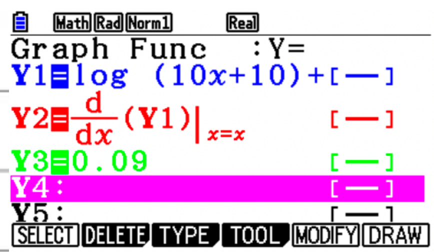
as OPTN-CALC is already open, press d/dx F2

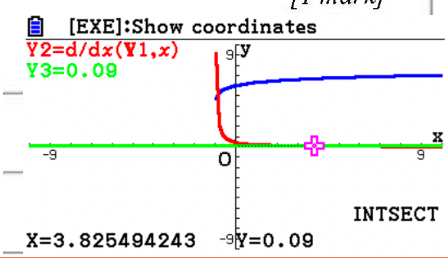


$H'(1.5) = 0.1737$ (1)

e) Determine the training time (as a decimal) when the rate of change in kick height is 0.09 m/day. *[1 mark]*

In graph app
1. Plot $H(t)$
2. Plot $H'(t)$
OPTN, CALC F2, d/dx F1
Use F1 to enter Y1
3. Plot $y=0.09$
4. Find intersection of derivative & $y=0.17$





$\therefore H'(t) = 0.09$

$\Rightarrow t = 3.825$ days (1)

Do not write outside this box.

0053GZ0Q208

QUESTION 15 **(4 marks)**

The term *extremely tall* is used to describe any person whose height is three standard deviations or more above the mean height of the population.

A person who just qualifies as extremely tall in a country where heights are normally distributed with a mean height of 180 cm and a standard deviation of 10 cm travels to another country. The person discovers they are taller than exactly 90% of the destination country's population.

Assuming that the standard deviation of both countries is the same, determine the minimum height required to be considered extremely tall in the destination country.

Let H_o = heights of people in original country

$H_o \sim N(180, 10^2)$

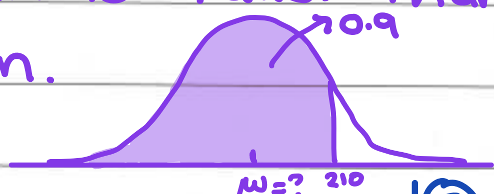
If they just qualify as extremely tall:

$\Rightarrow$ height $= 180 + 3(10) = 210$cm (1)

Let H_d = heights of people in destination country

The person is taller than 90% of th population.

$H_d \sim N(\mu, 10^2)$



① Calc z-score for 90% area

Statistics app

dist F5, NORM F1, InvN F3

Inverse Normal
Data :Variable
Tail :Left
Area :0.9
σ :1
μ :0
Save Res:None
None LIST

Inverse Normal
xInv=1.28155157

$z = 1.28155$ (1)

② $\therefore z = \dfrac{x-\mu}{\sigma}$

$1.28155 = \dfrac{210-\mu}{10}$

$12.8155 - 210 = -\mu$

$\therefore \mu = 197.1845$ (1)

$\therefore$ cut off for extremely tall in destination country

$= 197.1845 + 3(10)$

$= 227.1845$m (1)

Do not write outside this box.

0053GZ0Q209

QUESTION 16 (4 marks)

At council meetings in a particular town, new proposals are only discussed if more than 80% of the community are in favour of the proposal.

To discover community opinion on a new bus route proposal, the council conducted several surveys, each with a sample size of 120. The distribution of the sample proportions from the surveys had a standard deviation of 0.04.

Make a justified decision as to whether the new bus route proposal would be discussed at a council meeting.

$\sigma = \sqrt{\dfrac{p(1-p)}{n}}$ $\sigma = 0.04$

$n = 120$

$0.04 = \sqrt{\dfrac{p(1-p)}{120}}$ (1)

$\Rightarrow 0.04^2 = \dfrac{p(1-p)}{120}$

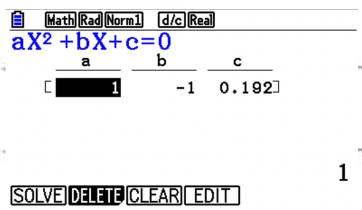
$\Rightarrow 120 \times 0.04^2 = p(1-p)$

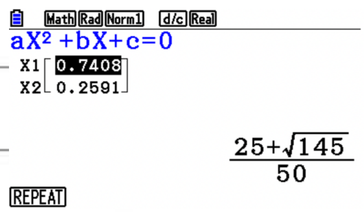
$\Rightarrow p^2 - p + 120 \times 0.04^2 = 0$

Equation app

POLY F2

2 F1 (quadratic)





$\therefore p = 0.7408$ or 0.2591 (2)

Both proportions are less than 80%, thus the council won't discuss the new proposal. (1)

Do not write outside this box.

0053GZ0Q210

QUESTION 17 (3 marks)

At a particular orchard, 3% of fruit is bruised during picking. After picking, the fruit is packed into boxes, each containing four pieces of fruit.

A grocery shop orders 140 boxes of fruit to sell to their customers.

Determine the expected number of boxes that will contain bruised fruit.

Let B = # of boxes with bruised fruit

$E[B] = np$

$n = 140$

We need to determine probability of a box containing bruised fruit.

$P(\text{at least 1 bruised})$

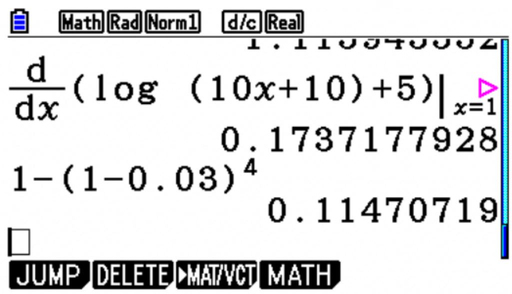
$= 1 - P(\text{no bruised})$

$= 1 - (1-0.03)^4$

$= 0.1147$ (1)

note: BCD could also be used here.

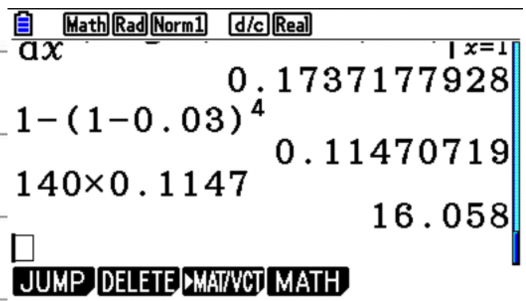
In the Run mode:



$\therefore E[B] = 140 \times 0.1147$

$= 16.058$ (1)

In the run mode:



The expected number of boxes with bruised fruit is 16.

Do not write outside this box.

QUESTION 18 (5 marks)

An object experiencing straight-line motion along a path has an acceleration (m s^{-2}) defined by the function $a(t) = 3\sin(2t)$ where t is time (s) since the object begins moving, $t \geq 0$.

When $t = 0$, both displacement and velocity are zero.

On the path is a motion sensor that is able to detect motion up to 2 metres away.

The object passes directly by the motion sensor when $t = 3$.

Determine the average velocity of the object while it moves through the range of the sensor.

$a(t) = 3\sin(2t) \quad t \geq 0$

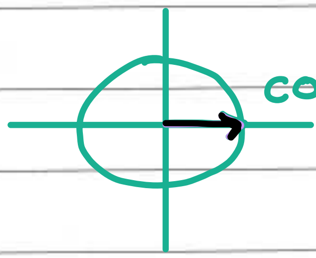
When $t = 0$, displacement & velocity $= 0$

$v(t) = \int 3\sin(2t)\,dt$ $\quad v(t) = \int a(t)\,dt$

$= -\frac{3}{2}\cos(2t) + c$

at $t = 0$, $v = 0$

$\therefore \quad 0 = -\frac{3}{2}\cos(0) + c$



$\cos(0) = 1$

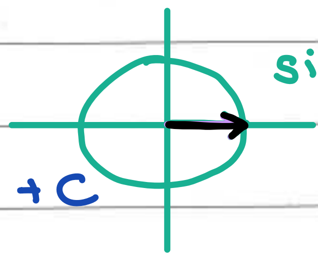
$c = +3/2$

$\therefore v(t) = -\frac{3}{2}\cos(2t) + \frac{3}{2}$ $\quad (1)$

$s(t) = \int -\frac{3}{2}\cos(2t) + \frac{3}{2}\,dt$ $\quad s(t) = \int v(t)\,dt$

$= -\frac{3}{4}\sin(2t) + \frac{3}{2}t + c$

$\sin(0) = 0$



at $t = 0$, $s = 0$

$0 = -\frac{3}{4}\sin(0) + \frac{3}{2}(0) + c$

$\therefore c = 0$

$\therefore\ s(t) = -\frac{3}{4}\sin(2t) + \frac{3}{2}t$ (1)

The object passes the sensor at $t=3$

$\therefore\ s(3) = -\frac{3}{4}\sin(2\times3) + \frac{3}{2}(3) = 4.70956$ (1)

In the run mode:

* calculator in radians!

```
Math Rad Norm1  d/c Real
                    0.11470719
140×0.1147
                        16.058
-3/4 sin (2×3)+3/2 (3)
                   4.709561624
JUMP DELETE MAT/VCT MATH
```

The motion sensor can detect movement from 2 meters away

$\Rightarrow$ can detect between $s_1 = 4.70956 - 2 = 2.70956$

& $s_2 = 4.70956 + 2 = 6.70956$

Need to find times when s_1 & s_2

i.e. $2.70956 = -\frac{3}{4}\sin(2t_1) + \frac{3}{2}t_1 \Rightarrow t_1 = 1.6891$

$6.70956 = -\frac{3}{4}\sin(2t_2) + \frac{3}{2}t_2 \Rightarrow t_2 = 4.5921$

Use solvN in run mode:

```
Math Rad Norm1  d/c Real
                    4.709561624
SolveN(2.70956=-3/4 sin▷
             {1.689135701}
SolveN(6.70956=-3/4 sin▷
             {4.592136521}
Solve d/dx d²/dx² ∫dx SolveN ▷
```

To save time, instead of typing twice, try editing the eq^n using the up & left keys.

① $\therefore$ The object is seen by sensor for $t_2 - t_1 = 2.903$ s (1)

② $\therefore$ average velocity $= \frac{s}{t} = \frac{4}{2.903} = 1.377$ m/s (1)

motion is captured for 4m

Do not write outside this box.

0053GZ0Q213

QUESTION 19 (6 marks)

The normal distribution probability density function is

$$p(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$, with the parameters mean, μ, and standard deviation, σ.

The speeds of electric scooter (e-scooter) riders on a particular section of a bike path are approximately normally distributed with a mean of 18 km/h. It is known that $p(10) = 0.0135$.

The speed limit for e-scooters on this section of bike path is 23 km/h.

A speed camera is set up and records the speeds of 75 e-scooter riders. Every rider travelling faster than the speed limit is given a $143 fine. Before setting up the speed camera, the following suggestion was made.

The total of the fines expected to be issued will be more than $1500.

Evaluate the reasonableness of this suggestion.

approx $\frac{1500}{143} \approx 10$ cars

Let s = speed of e-scooter

$S \sim N(18, \sigma^2)$ & $p(10) = 0.0135$

Lets find σ: (1)

$$0.0135 = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{10-18}{\sigma}\right)^2}$$

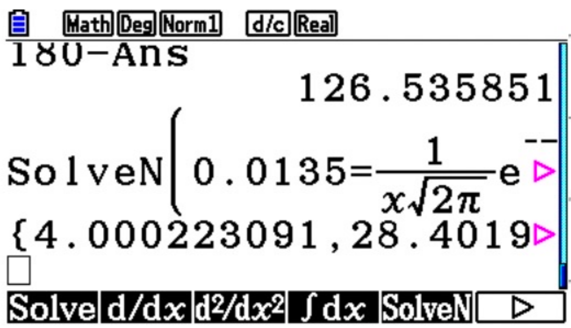
Solve using solveN

OPTN,

CALC F4,

SolveN F5

*use x to represent σ



(1)

$\therefore \sigma = 4.00$ or $\sigma = 28.40$

$\sigma = 28.40$ is not a possible standard deviation as it would result in negative speeds:

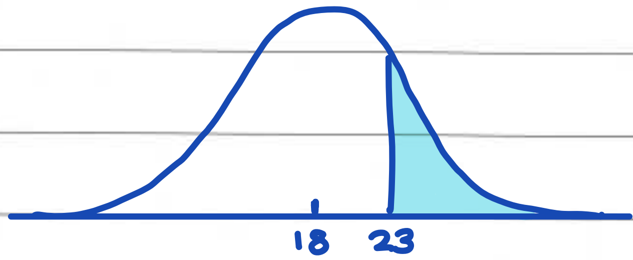
i.e. $18 - 1(28.4) = -10.4 < 0$

Do not write outside this box.

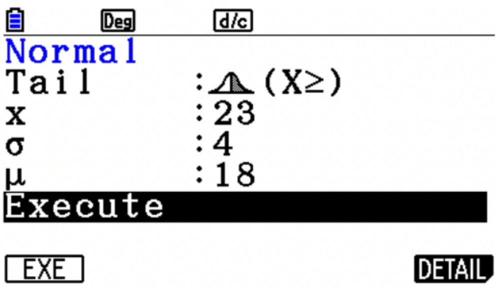
0053GZ0Q214

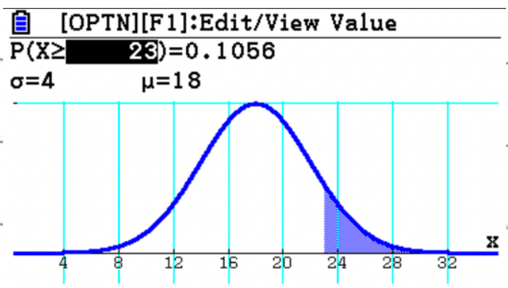
We need to determine the probability of exceeding a speed of 23km/hr (and hence getting a fine).

$S \sim N(18, 4^2)$



Using the distribution app:





$\therefore P(S \geqslant 23) = 0.1056$ (1)

The expected # of riders who exceed the speed limit in a sample of 75 riders is $0.1056 \times 75 = 7.92$. (1)

$\therefore$ Cost of fines $= 7.92 \times 143 = \$1132.56$ (1)

$< \$1500.00$

$\therefore$ The claim is not reasonable. (1)

END OF PAPER

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561001785

External assessment 2024

Multiple choice question book

Mathematical Methods

Paper 1 — Technology-free

General instruction

- Work in this book will not be marked.

Queensland Government | QCAA Queensland Curriculum & Assessment Authority

Section 1

Instruction

- Respond to these questions in the question and response book.

QUESTION 1

Determine $\int x^4 dx$

(A) $4x^3 + c$

(B) $5x^5 + c$

(C) $\frac{1}{3}x^3 + c$

(D) $\frac{1}{5}x^5 + c$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + c$$

$$\int x^4 \, dx = \frac{1}{5} x^5 \underline{\underline{+c}}$$

don't forget the +c

QUESTION 2

Determine $\frac{dy}{dx}$ for the function $y = e^{\sin(x)}$

(A) $\cos(x)\, e^{\sin(x)}$

(B) $\sin(x)\, e^{\cos(x)}$

(C) $e^{\sin(x)}$

(D) $e^{\cos(x)}$

$$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x) e^{f(x)}$$

$$\therefore \frac{dy}{dx} = \cos(x) e^{\sin(x)}$$

QUESTION 3

A sample of size n can be used to obtain a sample proportion $\hat{p}$.

An approximate margin of error for the population proportion can be obtained using the formula

$$E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

If the level of confidence is increased from 95% to 99%, then

(A) the associated z-value would decrease, so E would increase.

(B) the associated z-value would increase, so E would increase.

(C) the associated z-value would decrease, so E would decrease.

(D) the associated z-value would increase, so E would decrease.

• For 95% CI,
$Z = 1.96$
• For 99% CI,
$Z = 2.576$
$\Rightarrow$ increase
$\therefore$ multiplying by a larger number means E would also increase
$\Rightarrow$ B

QUESTION 4

Simplify $y = 2\ln\left(e^x\right)$

(A) $y = 2x$

(B) $y = 2^x$

(C) $y = \frac{2}{x}$

(D) $y = x^2$

$\ln e^x = x$

$\therefore y = 2\ln(e^x)$
$= 2x$

QUESTION 5

Determine $\int_a^b 2\cos(x)\,dx$, where $a = \frac{\pi}{3}$ and $b = \frac{\pi}{2}$

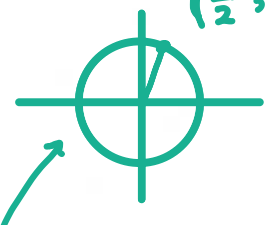
(A) $1 - \frac{\sqrt{3}}{2}$

(B) $\frac{\sqrt{3}}{2} - 1$

(C) $2 - \sqrt{3}$

(D) $\sqrt{3} - 2$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



$$\int_{\pi/3}^{\pi/2} 2\cos(x)\,dx$$
$$= \left[2\sin(x)\right]_{\pi/3}^{\pi/2}$$
$$= 2\sin\left(\frac{\pi}{2}\right) - 2\sin\left(\frac{\pi}{3}\right)$$
$$= 2(1) - 2\left(\frac{\sqrt{3}}{2}\right) = 2 - \sqrt{3}$$

QUESTION 6

Product rule: $u'v + uv'$

Differentiate $y = \ln(x)\cos(x)$ with respect to x.

(A) $\frac{\cos(x)}{x}$

(B) $-\frac{\sin(x)}{x}$

(C) $\frac{\cos(x)}{x} + \ln(x)\sin(x)$

(D) $\frac{\cos(x)}{x} - \ln(x)\sin(x)$

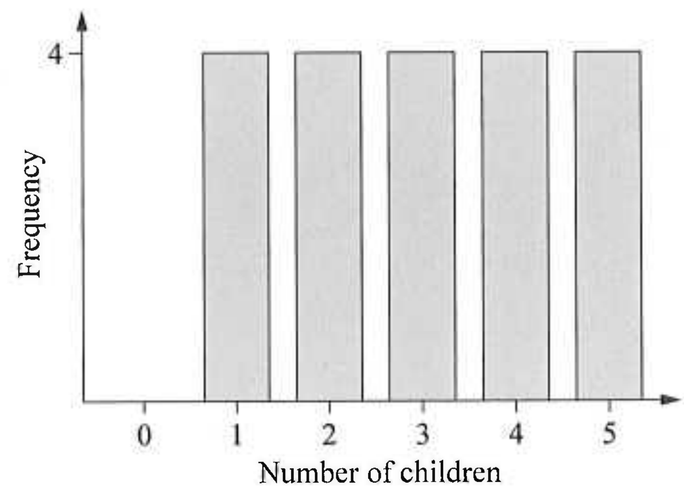
$$\frac{dy}{dx} = \frac{1}{x}(\cos x) + \ln(x)(-\sin x)$$

$$= \frac{\cos x}{x} - \ln x \sin x$$

QUESTION 7

Twenty families are selected to participate in a lifestyle study related to family size.

The number of children in these families is uniformly distributed as shown.



A random sample of five families is chosen from this group, without replacement. A possible mean number of children in the sample is

(A) 5.0

(B) 2.0

(C) 1.0

(D) 0.0

consider smallest # of children in a random sample of 5 families:

4 × 1 child
1 × 2 children $\Rightarrow$ mean $= \frac{4\times1+1\times2}{5} = \frac{6}{5}$

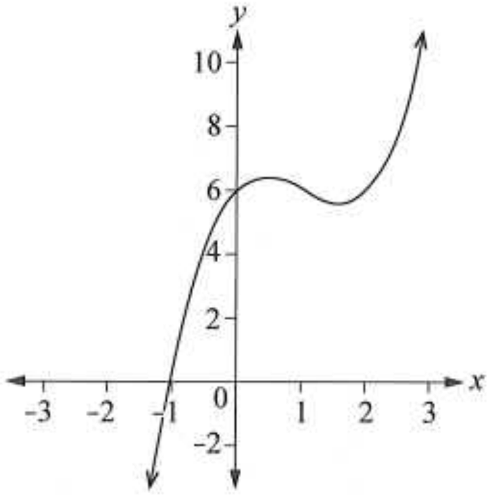
consider largest # of children in a random sample of 5 families:

4 × 5 children
1 × 4 children $\Rightarrow$ mean $= \frac{4\times5+1\times4}{5} = \frac{24}{5}$

The mean number of children in a sample will lie between $\frac{6}{5} = 1.2$ & $\frac{24}{5} = 4.8$. We can thus rule out answers A, C & D as they do not lie in the above region & the answer is C.

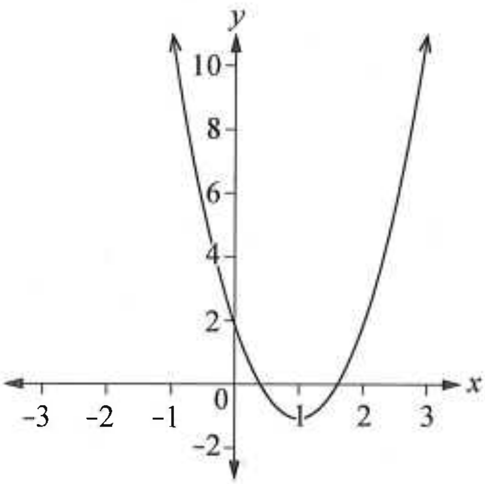
QUESTION 8

The graph of $f(x)$ is shown.

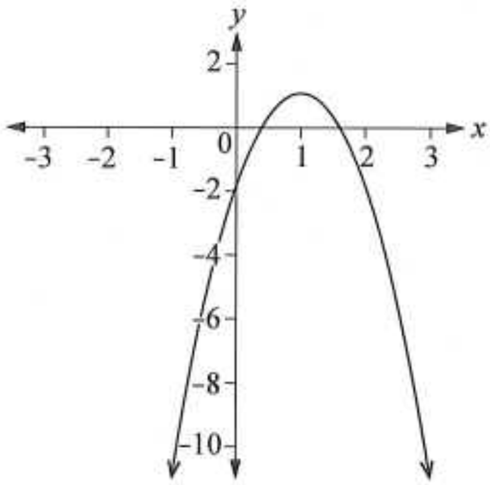


Identify the graph of the second derivative $f''(x)$.

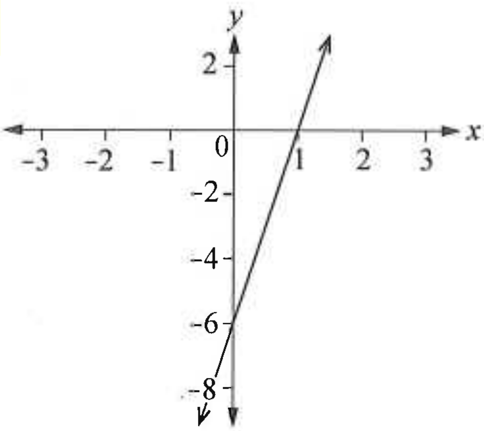
(A)



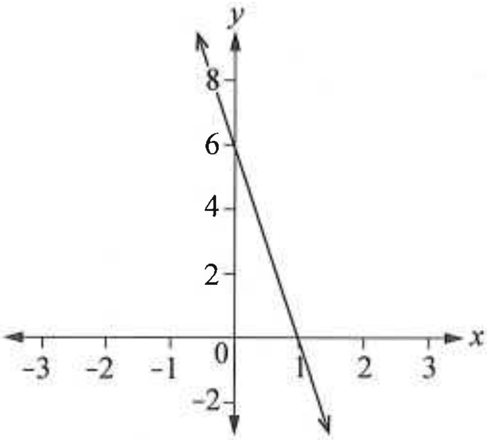
(B)



(C)



(D)



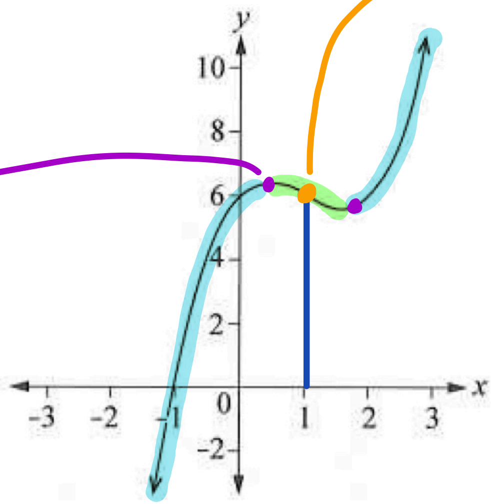
see next page for detailed soln

QUESTION 8

The graph of $f(x)$ is shown.

Method 1

$f''(1)=0$



Identify the graph of the second derivative $f''(x)$.

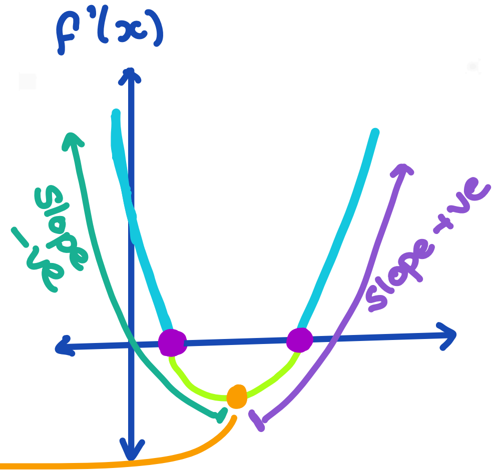
max / min $\Rightarrow f'(x)=0$

inflection pt $\Rightarrow$ max or min @ $f'(1)$ & $f''(x)=$

slope positive

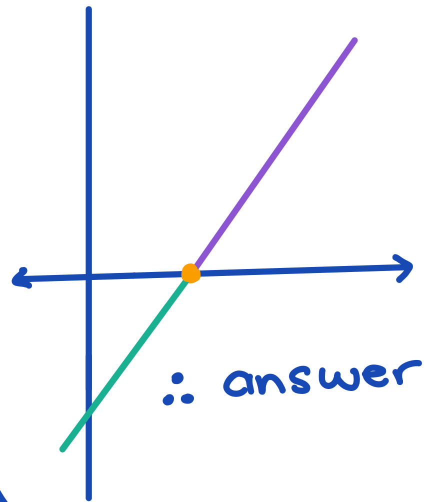
slope -ve

leads to :



$f''(1)=0$

$\therefore f''(x)$ looks like



$\therefore$ answer is C.

Method 2

Consider the inflection point at $x=1$, $f''(1)=0$.

When $x<1$,

$f(x)$ is concave down $\cap$

$\Rightarrow f''(x)<0$ for $x<1$.

When $x>1$,

$f(x)$ is concave up $\cup$

$\Rightarrow f''(x)>0$ for $x>1$

The only function which demonstrates this is the function in option C. $\therefore$ answer is C.

QUESTION 9

At a certain location, the temperature (°C) can be modelled by the function $T = 5\sin\left(\frac{\pi}{12}x\right) + 23$, where x is the number of hours after sunrise.

Determine the rate of change of temperature (°C/hour) when $x = 4$

(A) $\frac{5\pi}{48}$

(B) $\frac{5\pi}{24}$

(C) $\frac{5\pi\sqrt{3}}{24}$

(D) $\frac{5\pi\sqrt{3}}{6}$

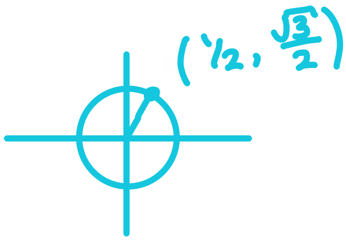
rate of change
=> derivative

$$\frac{dT}{dx} = \frac{5\pi}{12}\cos\left(\frac{\pi}{12}x\right)$$

$$\left.\frac{dT}{dx}\right|_{x=4} = \frac{5\pi}{12}\cos\left(\frac{\pi}{12}(4)\right)$$

$$= \frac{5\pi}{12}\cos\left(\frac{\pi}{3}\right)$$

$$= \frac{5\pi}{12} \times \frac{1}{2} = \frac{5\pi}{24}$$



QUESTION 10

Given that $\log_{10} 6 = 0.778$, determine the value of $\log_{10} 600$

(A) 77.800

(B) 10.778

(C) 2.778

(D) 1.556

$\log(AB) = \log(A) + \log(B)$

$$\log_{10}(600) = \log_{10}(6) + \log_{10}(100)$$

$$= 0.778 + 2$$

$$= 2.778$$

QUESTION 11 (6 marks)

a) Determine the second derivative of $y = x^3 - 3x^2$. *[2 marks]*

$y' = 3x^2 - 6x$ (1)

$y'' = 6x - 6$ (1)

b) Use your result from Question 11a) to calculate the value of the second derivative when $x = -1$. *[1 mark]*

$y''(-1) = 6(-1) - 6$

$= -12$ (1)

c) Determine the x- and y-coordinates of the point on the graph of $y = x^3 - 3x^2$ for which the rate of change of the first derivative is zero. *[3 marks]*

$\Rightarrow$ 2nd derivative $= 0$ * find both coordinates *

From part a, $y'' = 6x - 6$

$y'' = 0 \Rightarrow 0 = 6x - 6$ (1)

$6x = 6$

$x = 1$ (1)

$y(1) = 1^3 - 3(1)^2 = -2$ (1)

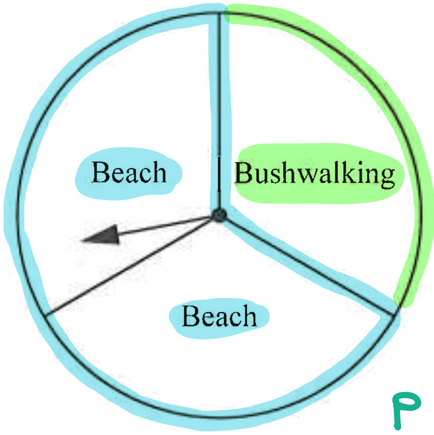
$\therefore$ point $= (1, -2)$

Do not write outside this box.

QUESTION 12 (6 marks)

Each day over a three-day long weekend, a family spins a pointer on a circular board to decide whether they will spend the day at the beach or bushwalking. The circular board consists of three equal sections.

Let b = event of going to beach $P(b) = \frac{2}{3}$



Let BW = event of going bushwalking $P(BW) = 1/3$

$P(x) = \binom{n}{x} p^x q^{n-x}$

a) Determine the probability that the family will spend all three days bushwalking. *[1 mark]*

$P(BW=3) = 1/3 \times 1/3 \times 1/3$

$= 1/27$ (1)

b) Determine the following binomial probabilities, expressed as fully simplified fractions.

i. Exactly two days will be spent at the beach. ⇒ 2 days @ beach & 1 day BW *[2 marks]*

$P(b=2) = \binom{3}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^1$ $\binom{3}{2} = 3$

$= 3 \times 4/9 \times 1/3$ (1)

$= 4/9$

ii. Fewer than three days will be spent at the beach. → 0 days at beach, 1 day at beach, 2 days at beach $= 1 - \text{prob}(3 \text{ days @ beach})$ *[3 marks]*

$P(b<3) = 1 - P(b=3)$ (1)

$= 1 - \left[\binom{3}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^0\right]$ $\binom{3}{3} = 1$ (1)

$= 1 - 8/27$

$= 19/27$ (1)

Do not write outside this box.

QUESTION 13 (6 marks)

a) $F(x)=\int\left(4e^{2x}+\sin(2x)\right)dx$. Use integration to determine $F(x)$, if $F(0)=5$. *[3 marks]*

$F(x) = \frac{4}{2}e^{2x} - \frac{1}{2}\cos(2x) + c$ (1)

$F(0) = 5$

$\Rightarrow 5 = 2e^{0} - \frac{1}{2}\cos(0) + c$

$5 = 2 - \frac{1}{2} + c$

$c = \frac{7}{2}$ (1)

$\therefore F(x) = 2e^{2x} - \frac{1}{2}\cos(2x) + \frac{7}{2}$ (1)

b) If $\frac{dy}{dx}=\left(\frac{3x^7-2x}{x^4}\right)^2$, determine y. *[3 marks]*

need to simplify before integrating to get y

$\frac{dy}{dx} = \frac{(3x^7-2x)^2}{(x^4)^2}$

$= \frac{9x^{14} - 6x^8 - 6x^8 + 4x^2}{x^8}$

$= \frac{9x^{14} - 12x^8 + 4x^2}{x^8}$ (1)

$= 9x^6 - 12 + 4x^{-6}$ (1)

$\int \frac{dy}{dx}dx = \int 9x^6 - 12 + 4x^{-6}\,dx$

$y = \frac{9}{7}x^7 - 12x - \frac{4}{5}x^{-5} + C.$ (1)

Do not write outside this box.

QUESTION 14 (5 marks)

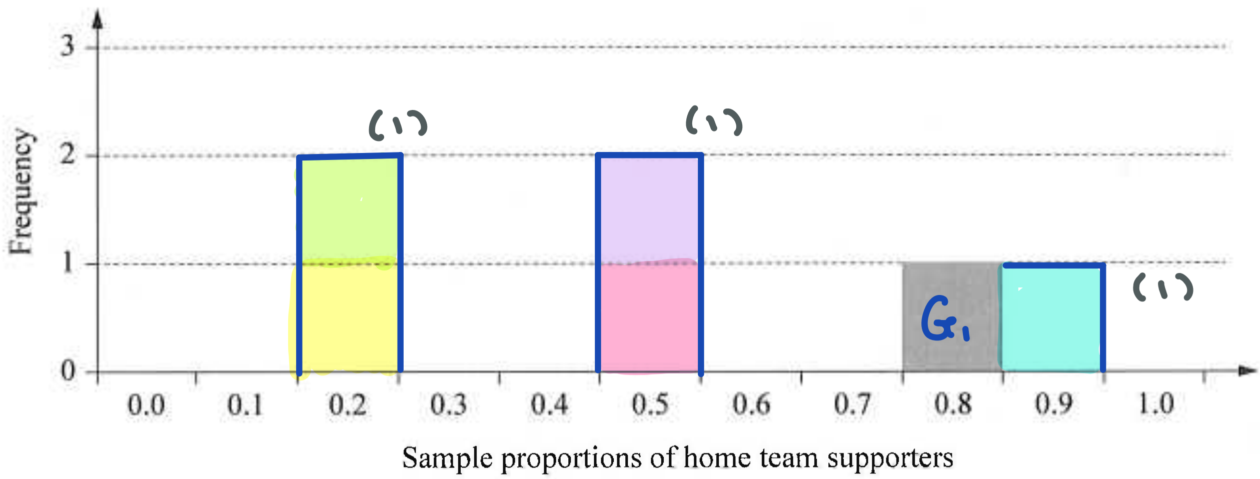
At a particular game at a local sporting venue, 60% of spectators support the home team and the remainder support the away team.

A researcher asked six groups of 10 spectators which team they supported. Each spectator was recorded as either H (supports home team) or A (supports away team). The results were:

Group 1:	H	A	H	H	H	A	H	H	H	H	
Group 2:	A	A	H	A	A	H	A	A	A	A	$\hat{p}_H = 0.2$
Group 3:	H	A	A	H	H	A	A	A	H	H	$\hat{p}_H = 0.5$
Group 4:	H	H	H	H	H	A	H	H	H	H	$\hat{p}_H = 0.9$
Group 5:	A	A	H	H	A	H	H	A	H	A	$\hat{p}_H = 0.5$
Group 6:	A	H	A	A	A	A	A	H	A	A	$\hat{p}_H = 0.2$

a) The researcher would like to see the distribution of the sample proportions of home team supporters obtained by entering the information into a column graph. The sample proportion for group 1 is shown.

Complete the column graph by including the sample proportions for the remaining five groups. *[3 marks]*



Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional response space at the back of this question and response book.

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b) If the researcher had interviewed more groups, each containing 100 spectators, describe two ways the distribution of the resulting sample proportions would be expected to differ from the distribution shown in Question 14a). *[2 marks]*

For larger samples, the distribution of $\hat{p}$ will become more symmetrical in Shape and closer approximate the normal distribution. (1)

For larger samples, the standard deviation is smaller leading to less variability about the mean. As a result, the graph will look narrowier. (1)

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QUESTION 15 (4 marks) – Method 1

A survey was conducted to understand whether people support a new policy.

Using a z-score of 2, the approximate confidence interval for the population proportion of people who support the policy was calculated as $\left(\frac{3}{10}, \frac{7}{10}\right)$.

a) Determine the margin of error. *[1 mark]*

$CI = (\hat{p} - E, \hat{p} + E)$ where E = margin of error

$\hat{p} - E = 3/10 \qquad \hat{p} + E = 7/10$

Subtracting the above 2 eq^{ns} (to eliminate $\hat{p}$)

$2E = 4/10$

$E = \frac{4}{10} \times \frac{1}{2}$

$E = \frac{1}{5}$ (1)

b) Determine the number of people surveyed. *[3 marks]*

$CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$. Find $\hat{p}$ and hence n.

1

$\hat{p} - E = 3/10$

$\therefore \hat{p} - \frac{1}{5} = \frac{3}{10}$

$\hat{p} = \frac{1}{2}$ (1)

2

$\therefore$ using upper CI

$\frac{7}{10} = \frac{1}{2} + 2\sqrt{\frac{\frac{1}{2}(1-\frac{1}{2})}{n}}$ (1)

$\frac{1}{5} = 2\sqrt{\frac{\frac{1}{2}(1-1/2)}{n}}$

$\frac{1}{10} = \sqrt{\frac{1/2(1-1/2)}{n}}$

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$$\frac{1}{100} = \frac{1/4}{n}$$

$$\therefore \frac{n}{1/4} = 100$$

$$n = \frac{100}{4}$$

$$n = 25$$ (1)

$\therefore$ 25 people were surveyed.

Do not write outside this box.

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QUESTION 15 (4 marks) - Method 2

A survey was conducted to understand whether people support a new policy.

Using a z-score of 2, the approximate confidence interval for the population proportion of people who support the policy was calculated as $\left(\frac{3}{10}, \frac{7}{10}\right)$.

a) Determine the margin of error. *[1 mark]*

margin of error $= \frac{\text{CI width}}{2}$

$= \frac{(\text{upper CI} - \text{lower CI})}{2}$

$= \frac{(0.7 - 0.3)}{2}$

$= 0.2$

b) Determine the number of people surveyed. *[3 marks]*

$\hat{p}$ = average of CI bounds

$= \frac{(\text{lower CI} + \text{upper CI})}{2}$

$= \frac{0.7 + 0.3}{2}$

$= 0.5$

∴ using upper CI

$\frac{7}{10} = \frac{1}{2} + 2\sqrt{\frac{\frac{1}{2}(1-\frac{1}{2})}{n}}$

$\frac{1}{5} = 2\sqrt{\frac{\frac{1}{2}(1-1/2)}{n}}$

$\frac{1}{10} = \sqrt{\frac{1/2(1-1/2)}{n}}$

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$$\frac{1}{100} = \frac{1/4}{n}$$

$$\therefore \frac{n}{1/4} = 100$$

$$n = \frac{100}{4}$$

$$n = 25$$

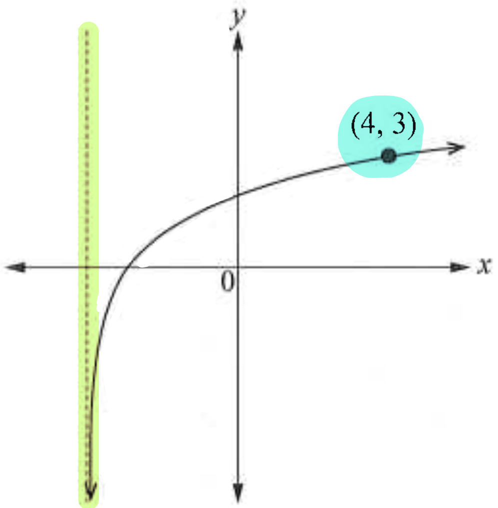
$\therefore$ 25 people were surveyed.

Do not write outside this box.

QUESTION 16 (4 marks)

The graph is of the form $y = \log_a(x+b)$. A point on the graph (4, 3) is labelled. The line $x = -4$ is an asymptote.

Note:
There are 3 key pieces of information you need to use to get to the answer



There is a point $P(x_P, y_P)$ on the graph where y_P is twice the value of the y-intercept of the curve.

Determine the value of x_P.

In the function $y=\log_a(x+b)$,
the asymptote occurs at $x+b=0$
In the above f^n, the asymptote occurs
at $x=-4$ $\Rightarrow$ $-4+b=0$
$b=4$. (1)

We know the graph passes thru (4,3)
$\therefore$ We can work out a
$3=\log_a(4+4)$ $\therefore a=2$ (1)
$\therefore \log_a(8)=3$
$\Rightarrow a^3=8$

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0053GZ0Q112

$\therefore$ Function is

$$y = \log_2(x+4)$$

Now y int => $x = 0$

$$\therefore\ y = \log_2(4)$$
$$y = 2$$

$\therefore$ y_p = twice the value of the y-int on the curve

$\therefore\ y_p = 2(2) = 4$ (1)

Finding x_p:

$$4 = \log_2(x_p + 4)$$
$$2^4 = 2^{\log_2(x_p+4)}$$
$$16 = x_p + 4$$
$$\therefore\ x_p = 12$$ (1)

$\therefore$ The point is $(12, 4)$

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QUESTION 17 (3 marks)

A community group that uses social media created a new post on the internet on a day when they had 1000 members. The rate of change in their number of members (members/day) is given by $f'(t) = 3e^{0.5t}$, where t represents days after the new post.

Determine the time it will take for the community group to achieve seven times the initial number of members. Express your answer in the form $a\ln(b)$.

$f'(t) = 3e^{0.5t}$ ⟶ rate of change in # of members per day

integrate to get # of members per day

$f(t) = \int 3e^{0.5t}\,dt$

$= \frac{3}{0.5}e^{0.5t} + c$ $\quad \int e^{f(x)}dx = \frac{1}{f'(x)}e^{f(x)}$

$f(t) = 6e^{0.5t} + c$ (1)

1 1000 members initially

i.e. $f(0) = 1000$

$1000 = 6e^{0.5(0)} + c$

$\therefore 1000 = 6 + c$ $\{e^0 = 1\}$

$\therefore c = 994$ (1)

$\therefore f(t) = 6e^{0.5t} + 994$

Need to find t when there are $1000(7) = 7000$ people

2 $7000 = 6e^{0.5t} + 994$

$6006 = 6e^{0.5t}$

$\frac{6006}{6} = e^{0.5t}$

$1001 = e^{0.5t}$ $\{\ln e^x = x\}$

$0.5t = \ln(1001)$

$t = 2\ln(1001)$ days

✓ (1)

units!

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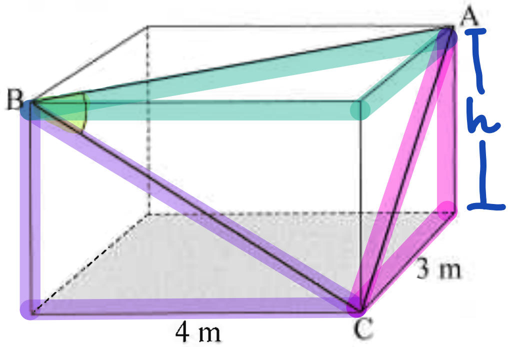
QUESTION 18 (5 marks)

The diagram shows some dimensions of a large storage container that is a rectangular prism. The angle ABC is 60°.

A person requires a container that is at least 4 metres in height.

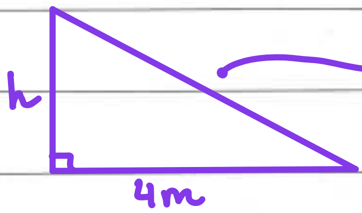
Not to scale



Make a justified decision about whether this storage container meets the person's requirements.

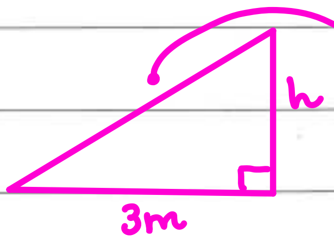
Let h = height of box

hmm, I wonder if we can work out the side lengths of ABC involving h...



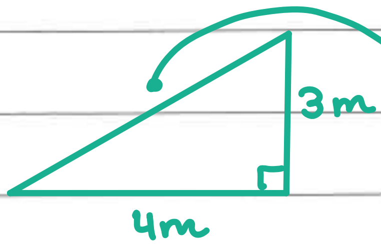
$\sqrt{h^2+4^2}$

$= \sqrt{h^2+16}$ (1)



$\sqrt{h^2+3^2}$

$= \sqrt{h^2+9}$ (1)



$\sqrt{3^2+4^2} = 5$

using pythag

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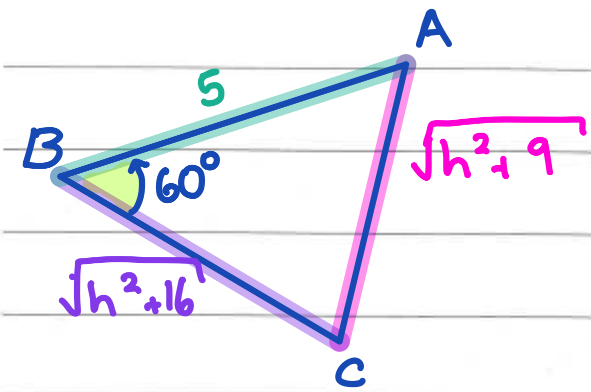

∴ We can use ΔABC & the cosine rule to solve for h

$$(\sqrt{h^2+9})^2 = (\sqrt{h^2+16})^2 + 5^2 - 2(\sqrt{h^2+16})(5)\cos(60°)$$

$$\cancel{h^2}+9 = \cancel{h^2}+16+25-2\sqrt{h^2+16}\,(5)\times \tfrac{1}{2}$$

①

$$-32 = -5\sqrt{h^2+16}$$

$$\therefore \sqrt{h^2+16} = \frac{32}{5}$$

$$h^2+16 = \left(\frac{32}{5}\right)^2$$

$$h^2 = \left(\frac{32}{5}\right)^2 - 16$$

$$h = \sqrt{\left(\frac{32}{5}\right)^2 - 16} \quad (1)$$

②

$$h = \sqrt{\left(\frac{64}{10}\right)^2 - 16}$$

$$h = \sqrt{6.4^2 - 16}$$

$$h > \sqrt{6^2 - 16}$$

Given $\sqrt{6^2-16} = \sqrt{20} > 4$

$\therefore h > 4$ and the container suits the person's requirements (1)

Do not write outside this box.

* (1) mark for logical organisation & showing key steps.

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QUESTION 19 (6 marks)

A permanent ice glacier is in a valley in New Zealand.

Due to the temperature changes of the seasons each year, the glacier expands for six months and recedes for six months. The changing distance of a point on the front edge of the glacier to a car park can be modelled by a sine function.

During the colder months, when the glacier expands, the front edge of the glacier moves to within 270 m of the car park. However, in the warmer months, when the glacier recedes, the front edge moves to a maximum distance of 280 m away from the car park.

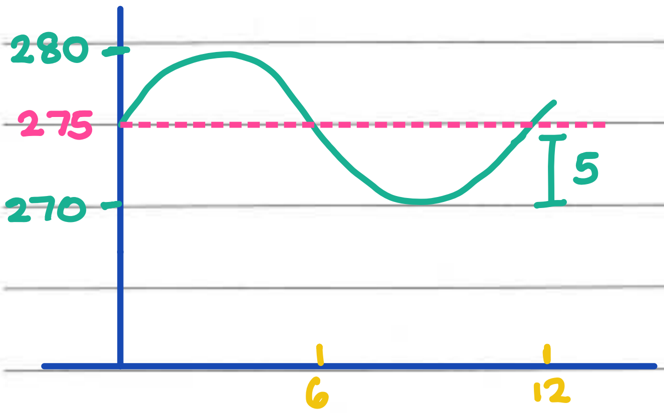
The erosion effects of the glacier on the ground are of most interest to geologists when the absolute value of the acceleration of the front edge is greater than $\frac{5\pi^2\sqrt{3}}{72}$ metres/month^2. During these times, a team of geologists sets up a camp site nearby to perform field work. Whenever the acceleration is less than this, the geologists leave camp.

The following claim is made.

The geologists will spend a total of between seven and eight months at the camp site each calendar year.

Evaluate the reasonableness of this claim.

The glacier's movement is modelled by a sine function: $G(x) = A\sin(B(x+C)) + D$



$A = 5$

period = 12 months

$\therefore B = \frac{2\pi}{12} = \frac{\pi}{6}$

$D = 275$

$C = 0$

$\therefore G(x) = 5\sin(\pi/6\, x) + 275$ (1)

Need to determine time periods when acceleration $> \frac{5\pi^2\sqrt{3}}{72}$ or $-\frac{5\pi^2\sqrt{3}}{72}$

absolute value

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$$G'(x) = \frac{5\pi}{6}\cos\left(\frac{\pi}{6}x\right) \quad (1)$$

$$G''(x) = -\frac{5\pi^2}{36}\sin\left(\frac{\pi}{6}x\right) \quad (1)$$

1

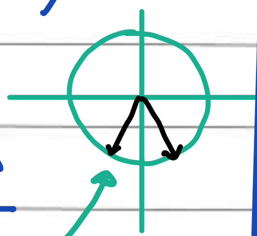
$$\frac{\cancel{5\pi^2}\sqrt{3}}{72} = -\frac{\cancel{5\pi^2}}{36}\sin\left(\frac{\pi}{6}x\right)$$

$$\frac{36\sqrt{3}}{72} = -\sin\left(\frac{\pi}{6}x\right)$$

$$-\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{6}x\right)$$

Let $X = \frac{\pi}{6}x$

$\therefore \sin(X) = -\frac{\sqrt{3}}{2}$



$$X = \frac{4\pi}{3} \text{ or } \frac{5\pi}{3}$$

$$\therefore x = \frac{6}{\pi}\times\frac{4\pi}{3} \text{ or } \frac{6}{\pi}\times\frac{5\pi}{3}$$

$x = 8$ or 10 months

2

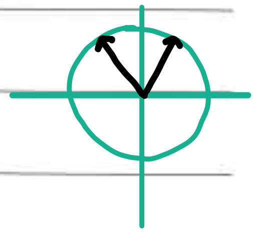
$$\cancel{-}\frac{\cancel{5\pi^2}\sqrt{3}}{72} = \cancel{-}\frac{\cancel{5\pi^2}}{36}\sin\left(\frac{\pi}{6}x\right)$$

$$\frac{36\sqrt{3}}{72} = \sin\left(\frac{\pi}{6}x\right)$$

$$\frac{\sqrt{3}}{2} = \sin\left(\frac{\pi}{6}x\right)$$

Let $X = \frac{\pi}{6}x$

$\sin(X) = \frac{\sqrt{3}}{2}$



$$X = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

$$\therefore x = \frac{6}{\pi}\times\frac{\pi}{3} \text{ or } \frac{6}{\pi}\times\frac{2\pi}{3}$$

$x = 2$ or 4 months

$G''(x)$ is a sine function and thus

$G''(x) > \frac{5\pi^2\sqrt{3}}{72}$ when $2 \le x \le 4$ (1) & $8 \le x \le 10$, (1)

i.e. 4 months of the year. (1) The claim is not reasonable as the period of time spent at the camp is 4 months not between 7 and 8 months. (1)