SUPERVISOR TO ATTACH
PROCESSING LABEL HERE

Write your **student number** in the boxes above. **Letter**

Mathematical Methods Examination 2

Question and Answer Book

VCE Examination – Thursday 6 November 2025

Unofficial Solutions

Errors, omissions and alternate methods may exist.

- Reading time is **15 minutes**: 11.45 am to 12 noon
- Writing time is **2 hours**: 12 noon to 2.00 pm

Additional notes (not necessarily part of the solution) in pink

Approved materials
- Protractors, set squares and aids for curve sketching
- One bound reference
- One approved CAS calculator or CAS software, and one scientific calculator

Notes on ClassPad use in green

Materials supplied
- Question and Answer Book of 28 pages
- Formula Sheet
- Multiple-Choice Answer Sheet

Instructions
- Follow the instructions on your Multiple-Choice Answer Sheet.
- At the end of the examination, place your Multiple-Choice Answer Sheet inside the front cover of this book.

Students are **not** permitted to bring mobile phones and/or any unauthorised electronic devices into the examination room.

Contents

VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

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Section A – Multiple-choice questions

Instructions

- Answer **all** questions in pencil on your Multiple-Choice Answer Sheet.
- Choose the response that is **correct** for the question.
- A correct answer scores 1; an incorrect answer scores 0.
- Marks will **not** be deducted for incorrect answers.
- No marks will be given if more than one answer is completed for any question.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

A function that has a range of [6, 12] is

~~A.~~ $f : R \to R,\ f(x) = 6 + 3\cos(9x)$

~~B.~~ $f : R \to R,\ f(x) = 6 + 6\cos(3x)$

C. $f : R \to R,\ f(x) = 9 - 3\cos(6x)$

D. $f : R \to R,\ f(x) = 9 - 6\cos(3x)$

P.A.: $y = \frac{6+12}{2} = 9$ $\therefore$ C or D

Amp.: $A = \frac{12-6}{2} = 3$ $\therefore$ C

Question 2

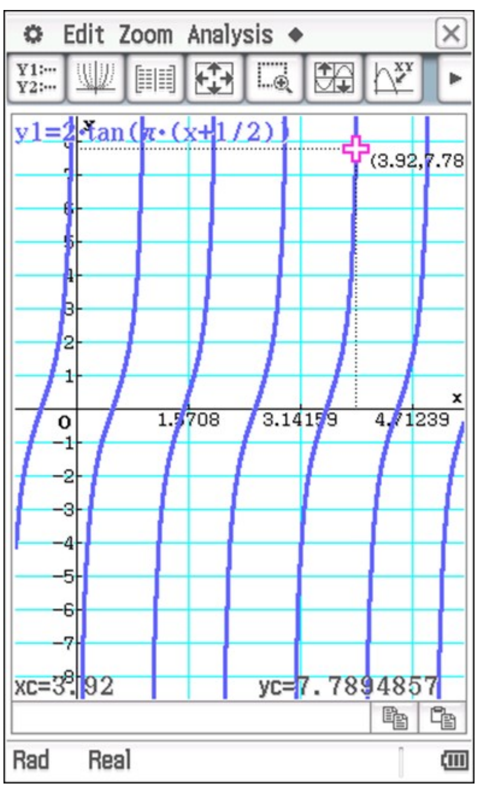
All asymptotes of the graph of $y = 2\tan\left(\pi\left(x + \frac{1}{2}\right)\right)$ are given by

A. $x = k,\ k \in Z$

B. $x = 2k,\ k \in Z$

C. $x = 2k + 1,\ k \in Z$

D. $x = \frac{4k+1}{2},\ k \in Z$



$y = \tan x$ has asymptotes at

$x = \frac{\pi}{2} + k\pi$ $(k \in \mathbb{Z})$

$\therefore$ Solve $\pi\left(x + \frac{1}{2}\right) = \frac{\pi}{2} + k\pi$

$\therefore$ $x = k \Rightarrow$ A

or

$y = 2\tan\left(\pi\left(x + \frac{1}{2}\right)\right)$ is $y = \tan x$

- dilated from the y-axis by a factor of $\frac{1}{\pi}$
- translated ½ a unit to the left

applying these to $x = \frac{\pi}{2} + k\pi$ we get

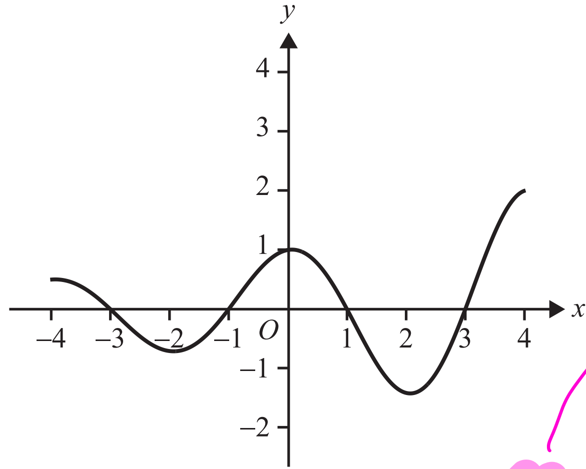
$\frac{1}{\pi}\left(\frac{\pi}{2} + k\pi\right) - \frac{1}{2} = \frac{1}{2} + k - \frac{1}{2} = \underline{\underline{k}}$ $\therefore$ A

or

- Draw a graph of $y = 2\tan\left(\pi\left(x + \frac{1}{2}\right)\right)$ in the Graph + Table app
- Tap Analysis → Trace
- use the left / right cursor to trace the graph, observing asymptotic behaviour at $x = 1, 2, 3$ etc

Question 3

The graph of $y = f(x)$ is shown below.

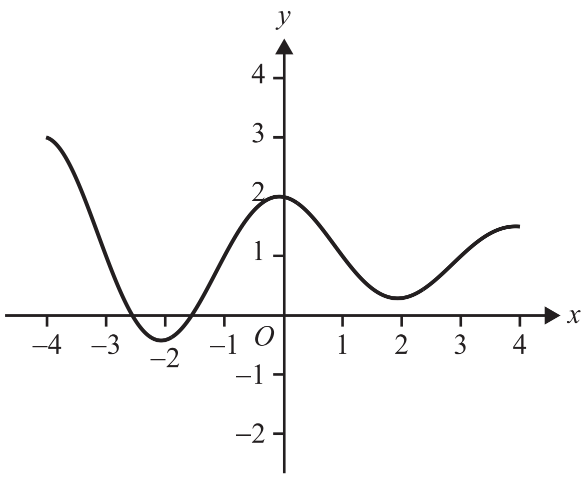


Which one of the following options best represents the graph of $y = f(-x) + 2$?

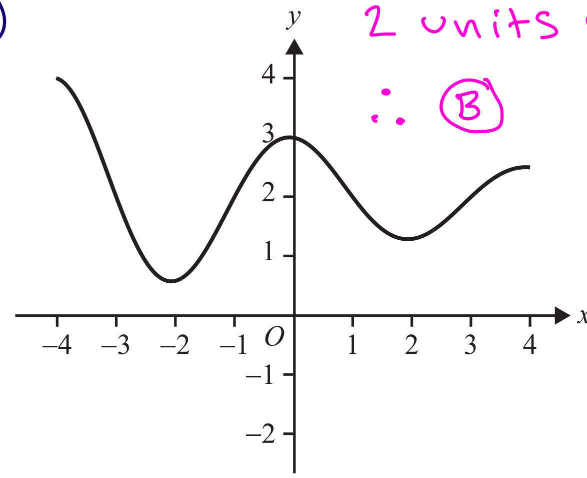
Reflected in the y-axis
$\therefore$ A or B

Translated 2 units up
$\therefore$ B

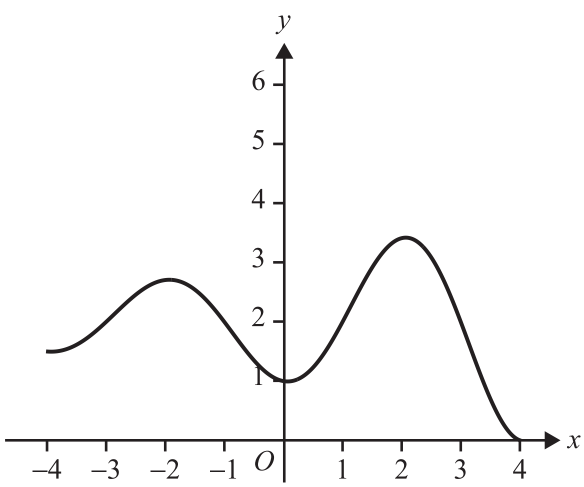
A.



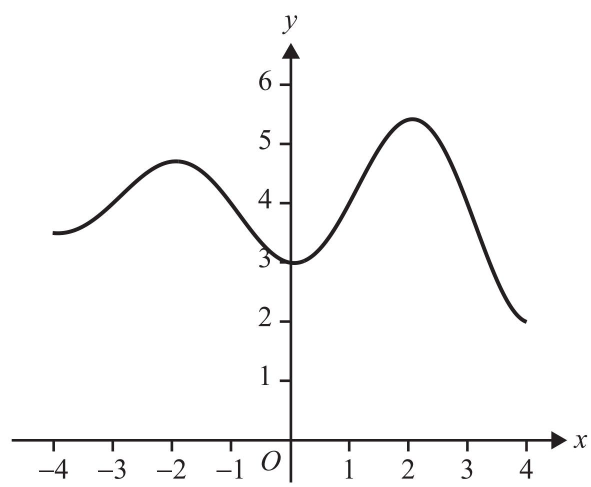
B.



~~**C.**~~



~~**D.**~~



Question 4

Consider the system of equations below containing the parameter k, where $k \in R$.

$$kx + 3y = k^2$$
$$2x + (2k+1)y = 6 - 2k$$

Find the value(s) of k for which this system has no real solutions.

A. $k = -2$ only

B. $k = \frac{3}{2}$ only

C. $k = -2$ or $\frac{3}{2}$

D. $k \in R \setminus \left\{-2, \frac{3}{2}\right\}$

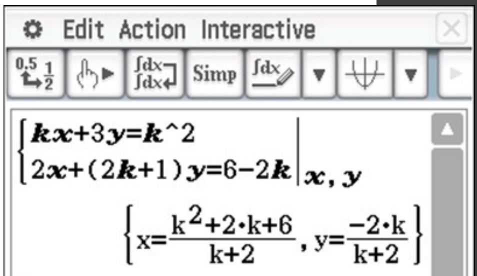
Infinite or no solution occur when

$-\frac{k}{3} = -\frac{2}{2k+1}$ (Same Slope)

$\therefore k = -2$ or $k = \frac{3}{2}$

$\frac{k^2}{3} = \frac{4}{3}$, $\frac{6-2k}{2k+1} = \frac{10}{-3}$ } diff. y-ints $\therefore$ no sol

$\frac{k^2}{3} = \frac{3}{4}$, $\frac{6-2k}{2k+1} = \frac{3}{4}$ } Same y-ints $\therefore$ inf sols



• In the Main App
• Use [ ] on the Math1 keyboard

Or

• Solve the system
• inspect the sols
• Sols are undefined if denom. = 0 i.e. if $k+2=0$
$\therefore k = -2$ (only)
$\therefore$ A

Question 5

Which of the following sets represents a function that has an inverse function?

A. $\{(1, 3), (2, 0), (2, 1)\}$

B. $\{(-1, 3), (2, 2), (3, 1)\}$

C. $\{(-1, 3), (0, 1), (1, 3)\}$

D. $\{(1, 0), (2, 3), (1, 3)\}$

A function $\Rightarrow$ no repeated x-values
$\therefore$ not A or D
Has an inverse $\Rightarrow$ no repeated y-values
$\therefore$ not C
$\therefore$ B

Question 6

The trapezium rule is used, with two trapeziums, to estimate the area bounded by the graph of $y = f(x)$, the x-axis and the lines $x = 0$ and $x = 1$.

For which function will the trapezium rule estimate be **larger** than the exact area?

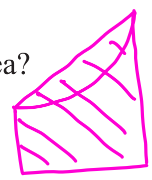
A. $f(x) = 3 - e^x$

B. $f(x) = x^3 + 1$

C. $f(x) = 3\sin(x) + 1$

D. $f(x) = \log_e(x+3)$

Trapezium > area $\Rightarrow$ concave up



Trapezium < area $\Rightarrow$ concave down

• A and D are concave down for all x

• C is concave down for $0 \le x \le \pi$

$\therefore$ B

Do not write in this area.

Question 7

Consider the algorithm below.

```
n ← 17
k ← 5
while n > k
    n ← n − k
    print n
end while
```

$n=17$, $17>5$ ∴ print $\underline{\underline{12}}$
$n=12$, $12>5$ ∴ print $\underline{\underline{7}}$
$n=7$, $7>5$ ∴ print $\underline{\underline{2}}$
$n=2$, $2<5$ ∴ end while

In order, the values printed by the algorithm are

A. 12
B. 12, 7
C. 12, 7, 2
D. 12, 7, 2, −3

Question 8

A random sample of n Victorian households is taken to estimate the proportion of all Victorian households that have vegetable gardens. The approximate 95% confidence interval calculated using this sample is (0.248, 0.552), correct to three decimal places.

The number of households, n, in the sample is

A. 10
B. 28
C. 40
D. 49

width is 0.304

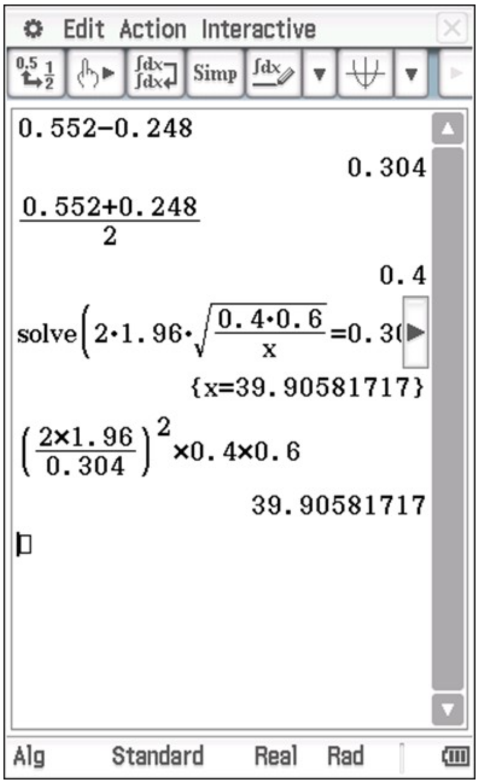
and $\hat{p} = 0.4$

$w = 2 \times 1.96 \times \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

∴ $n = 39.9\ldots$
∴ $n = 40$ ⇒ C

—— or ——

if you know this formula for sample size in terms of n



Question 9

One day, at a particular school, m students walked to school and the remaining n students travelled to school using a different form of transport.

Of the m students who walked, 20% took at least 30 minutes to get to school.

Of the n students who used a different form of transport, 40% took at least 30 minutes to get to school.

Given that a randomly selected student took at least 30 minutes to get to school, the probability that they walked to school is given by

A. $\dfrac{m}{m+2n}$

B. $\dfrac{2n}{m+2n}$

C. $\dfrac{m}{5(m+n)}$

D. $\dfrac{1}{3}$

$$\Pr(w \mid 30+) = \frac{\Pr(w \cap 30+)}{\Pr(w \cap 30+) + \Pr(\bar{w} \cap 30+)}$$

could use CP to simplify

$$= \frac{\frac{m}{m+n} \times 0.2}{\frac{m}{m+n} \times 0.2 + \frac{n}{m+n} \times 0.4} \times \frac{10(m+n)}{10(m+n)}$$

$$= \frac{2m}{2m+4n} = \frac{m}{m+2n} \quad \therefore A$$

Question 10

Consider $f: R \to R,\ f(x) = 2x^2 + x - 1$ and $g: R \to R,\ g(x) = \sin(x)$.

The inequality $(f \circ g)(x) > 0$ is satisfied when

A. $\sin(x) \le -1$

B. $-1 < \sin(x) < 0$

C. $\dfrac{1}{2} < \sin(x) \le 1$

D. $0 < \sin(x) < \dfrac{1}{2}$

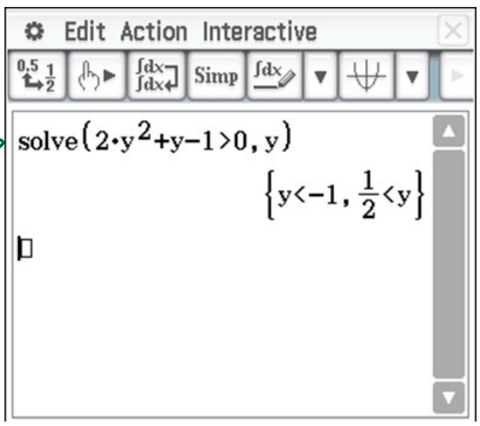
* $f(y) > 0$ where $y = \sin x$

$\therefore y < -1$ or $y > \frac{1}{2}$

and $-1 \le y \le 1$ as

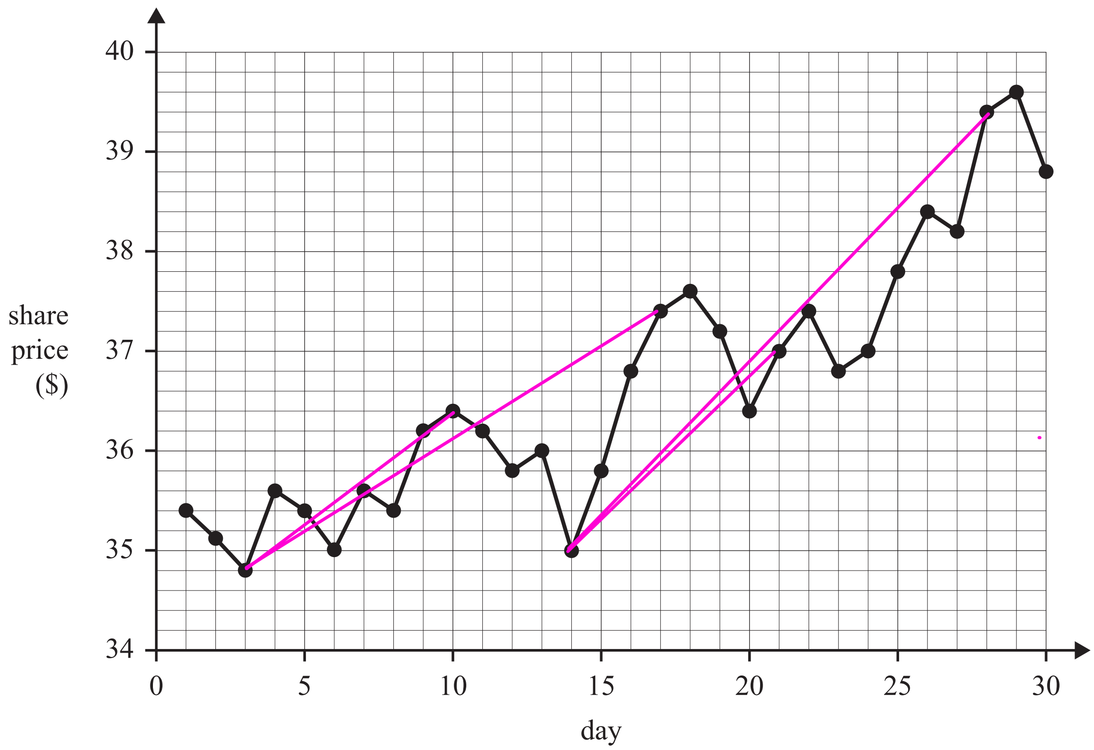
$\therefore \frac{1}{2} < y \le 1 \quad \therefore C$

* note - answers in terms of $\sin(x)$



Question 11

The chart below shows the daily price of a stock market share over a 30-day period.



Over which of the following time intervals did the daily price undergo the greatest average rate of change?

A. day 3 to day 10

B. day 3 to day 17

C. day 14 to day 21

D. day 14 to day 28

Use a ruler to compare gradients visually ∴ D

or

$\frac{\$1.60}{7} = 0.229$

$\frac{\$2.60}{14} = 0.186$

$\frac{\$2.00}{7} = 0.286$

$\frac{\$4.40}{14} = 0.314$ ∴ D

* routine classpad calculation

Question 12

For a normal random variable X, it is known that $\Pr(X > 200) = 0.325$ and $\Pr(180 < X < 200) = 0.589$

The mean and standard deviation of X are closest to

$x = 200 \Leftrightarrow z = 0.456$ $x = 180 \Leftrightarrow z = -1.366$

A. 190 and 10

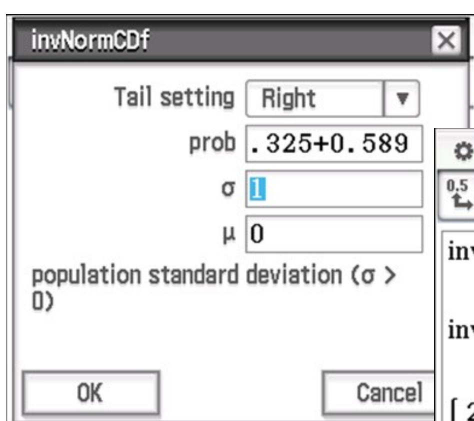
B. 190 and 11

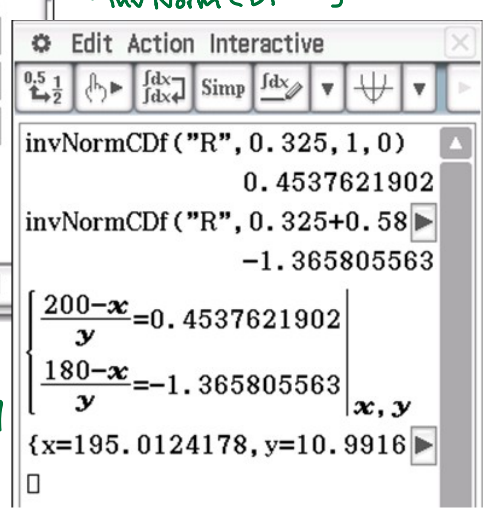
C. 195 and 10

D. 195 and 11

In the Main app, use
- Interactive
- Dist./Inv. Dist.
- Inverse
- inv Norm CDF

to calculate z-scores.



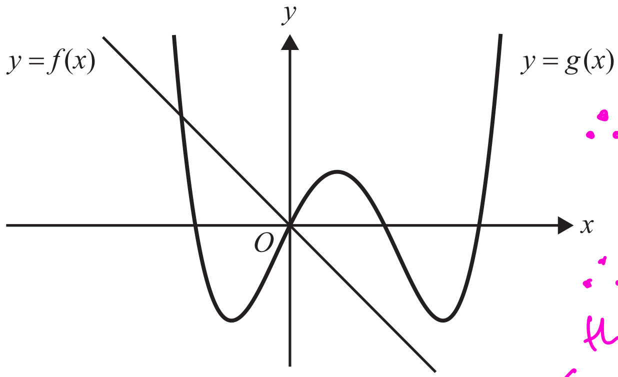


In Main App:
- Choose Sim. Equ. template from the Math1 keyboard

- Use the z-scores to generate and solve equations for mu & sigma (I used x & y)

Question 13

The graphs of $y = f(x)$ and $y = g(x)$ are sketched on the same set of axes below.



$f(x) = -kx$

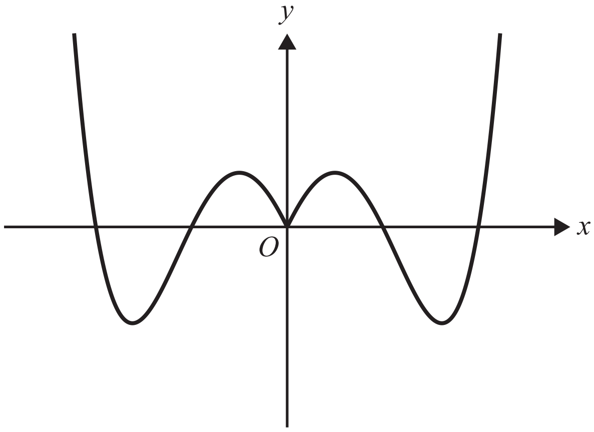
$\therefore g(f(x)) = g(-kx)$

$\therefore$ reflection in the y-axis $\therefore$ C

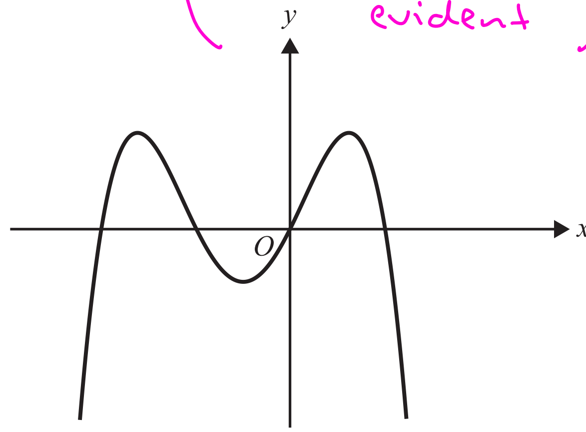
hor. dilation if $k \neq 1$, but none evident

Which of the following could be the graph of $y = (g \circ f)(x)$?

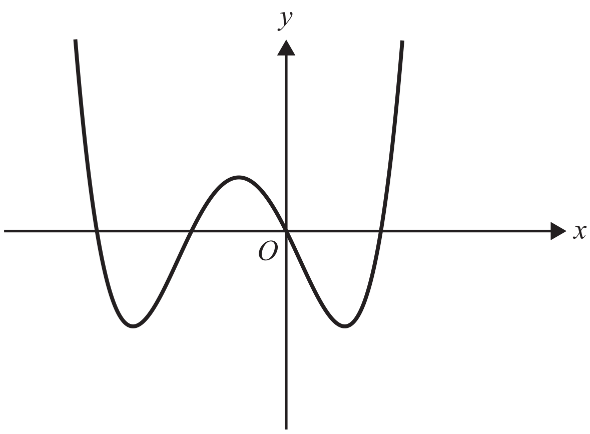
A.



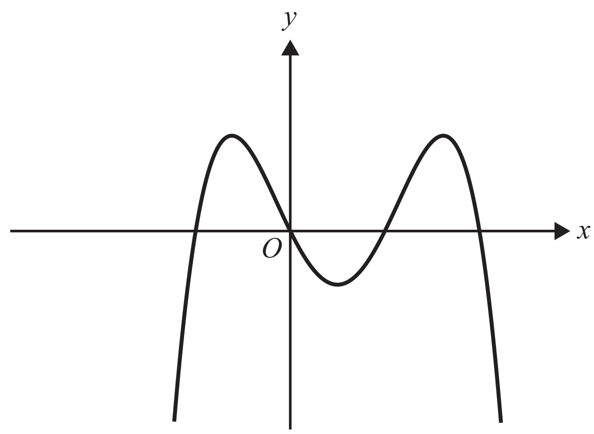
B.



C.



D.



Question 14

Let f be the probability density function for a continuous random variable X, where

$$f(x)=\begin{cases} k\sin(x) & 0 \le x < \frac{\pi}{4} \\ k\cos(x) & \frac{\pi}{4} \le x \le \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

and k is a positive real number.

The value of k is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2-\sqrt{2}}$

C. $\sqrt{2}+2$

D. $2-\sqrt{2}$

need to solve

$\int_0^{\pi/2} f(x)\,dx = 1$

or

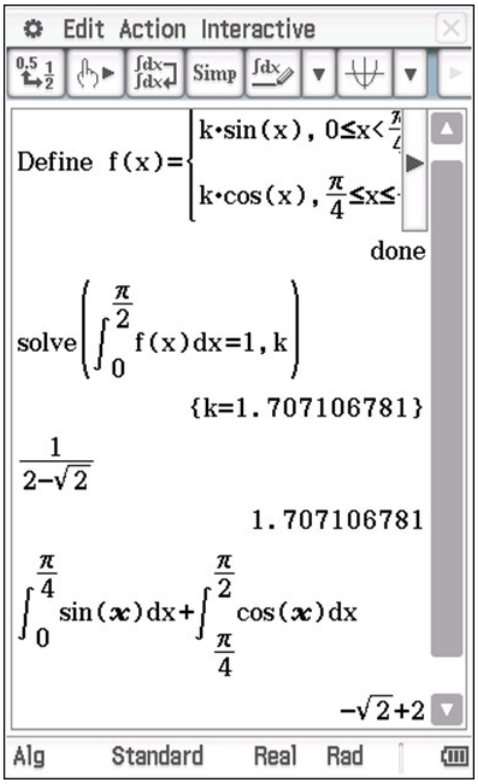
$\therefore k\int_0^{\pi/2} f(x)\,dx = 1$

$\therefore k = \frac{1}{\int_0^{\frac{\pi}{2}} f(x)\,dx}$

$\therefore$ B

• Defining a piece-wise function and solving an integral equation for k gives a decimal approx.

• Knowing that $\sqrt{2} = 1.4\ldots$ means opt. A < 1, C > 2, D < 1 $\therefore$ ans. is B (which can be checked)



Question 15

The graph of $y = g(x)$ passes through the point $(1, 3)$.

The graph of $y = 1 - g(2x-3)$ must pass through the point

A. $(-1, -2)$

B. $(2, -2)$

C. $(-1, 2)$

D. $(2, 2)$

we only have $g(1)$ defined

$\therefore 2x-3=1 \Rightarrow x=2$

@ $x=2$, $y = 1-g(1)$
$= 1-3 \Rightarrow (2,-2)$
$= -2$ $\therefore$ B

or

$y = -g\left(2\left(x-\frac{3}{2}\right)\right)+1$

• reflect $(1,3)$ in x-axis $(1,-3)$

• dilate factor $\frac{1}{2}$ from y gives $\left(\frac{1}{2},-3\right)$

• translate 1 unit up and $\frac{3}{2}$ units right gives $\left(\frac{1}{2}+\frac{3}{2}, -3+1\right) = (2,-2)$

$\therefore$ B

Question 16

Consider the function $h(x) = a\log_e(bx)$, where $a, b \in R \setminus \{0\}$.

Given that its derivative $h'(x)$ has range $(0, \infty)$, which of the following **must** be true?

A. $a > 0$ only

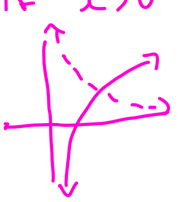
B. $a > 0$ and $b < 0$

C. $a > 0$ and $b > 0$

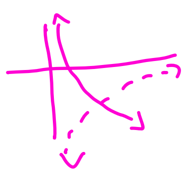
D. $ab > 0$

$h(x) = a\ln x(bx) \Rightarrow h'(x) = a \times \frac{b}{bx} = \frac{a}{x}$

if $x>0$ then $b>0$



$a>0$
$h'(x) \in (0,\infty)$ ✓



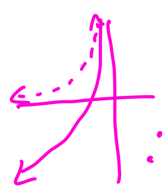
$a<0$
$h'(x) \in (0,-\infty)$
✗

$\therefore a>0$ and $b>0$

and

if $x<0$ then $b<0$

$a>0$ then
$h'(x) \in (0,-\infty)$
✗



$a<0$ then
$h'(x) \in (0,\infty)$ ✓

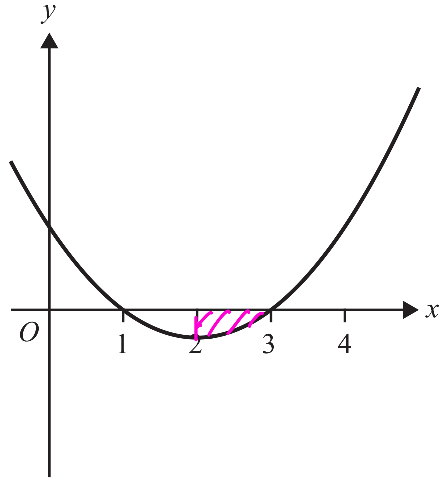
$\therefore a<0$ and $b<0$

So in both cases, $ab>0$ $\therefore$ D

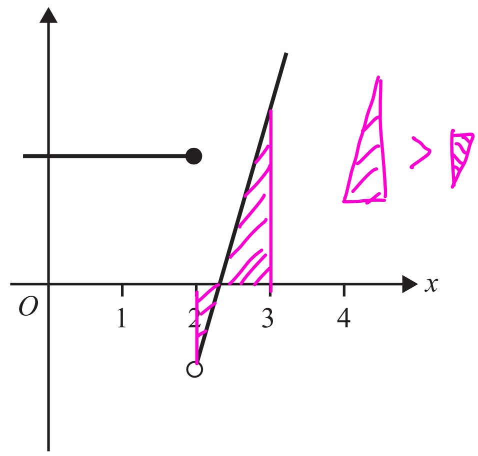
Question 17

Given that $f: R \to R$ satisfies $\int_1^2 f(x)\,dx > \int_1^3 f(x)\,dx$, the graph of $y = f(x)$ could be

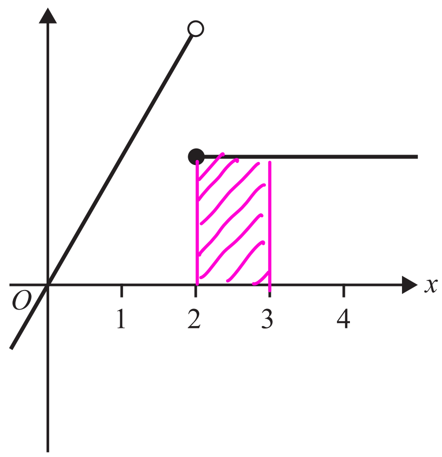
A.



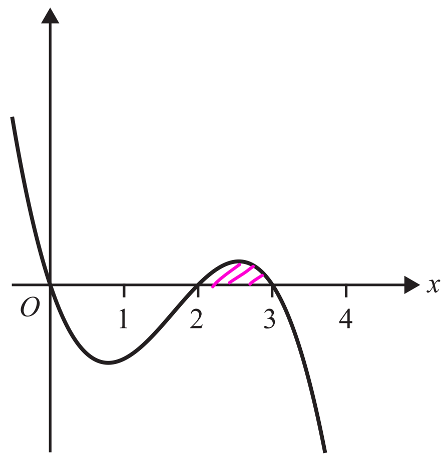
B.



C.



D.



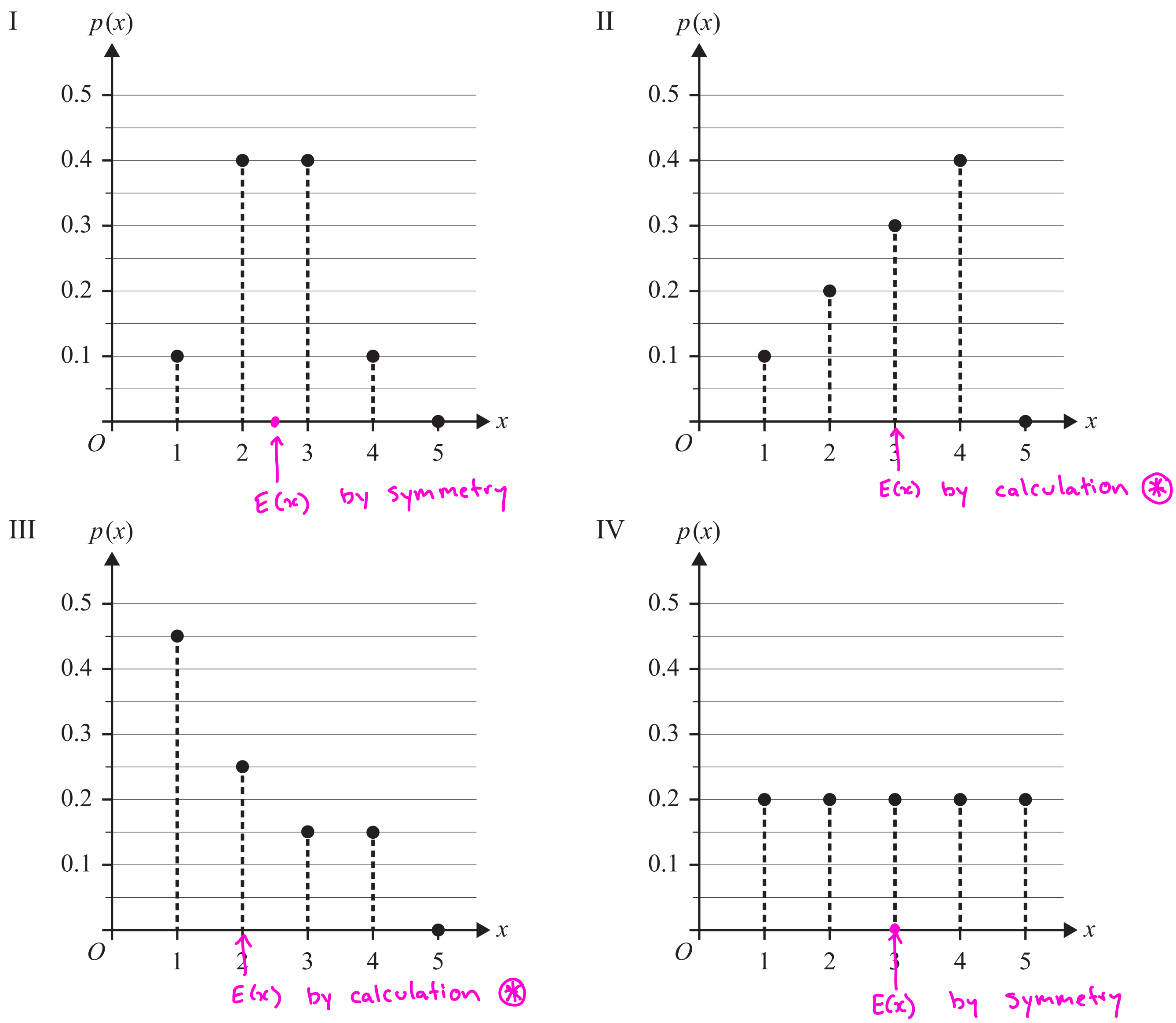
$$\int_1^2 f(x)\,dx > \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx$$

$$\therefore \int_2^3 f(x)\,dx < 0$$

considering shaded regions above, this is only true for A

Question 18

Consider the following graphs, which represent probability mass functions.



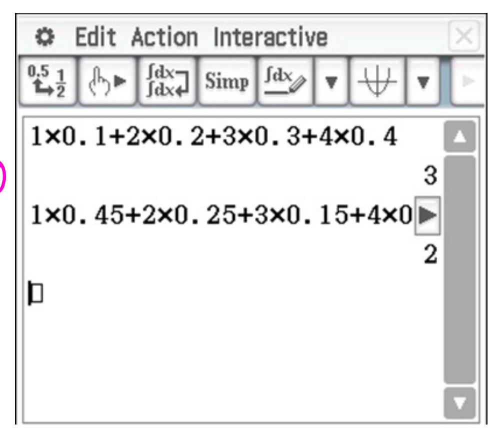
Which pair of these probability mass functions has the same mean?

A. I and II

B. I and IV

C. II and III

D. II and IV



Question 19

Let A be a point on the line $y = x + c$ and B be a point on the curve $y = \log_e(x-1)$.

If A and B are placed such that the line segment AB has the minimum possible length, and this length is $\sqrt{2}$, the value of c must be

* see next page

A. $\sqrt{2} - 2$

B. $\sqrt{2}$

C. 1

D. 0

Question 20

Let $a > 1$, and consider the functions f and g defined below.

$$f : R \to R,\ f(x) = a^x$$

$$g : R \to R,\ g(x) = a^{2x+2}$$

Which one of the following sequences of transformations, when applied to $f(x)$, does **not** produce $g(x)$?

A. dilation by a factor of $\frac{1}{2}$ from the y-axis, then translation by 1 unit in the negative direction of the x-axis

$a^x \to a^{2x} \to a^{2(x+1)} = g(x)$

B. dilation by a factor of $\frac{1}{2}$ from the y-axis, then dilation by a factor of a^2 from the x-axis

$a^x \to a^{2x} \to a^2 \times a^{2x} = g(x)$

C. dilation by a factor of a from the x-axis, then dilation by a factor of $\frac{1}{2}$ from the y-axis, then translation by 1 unit in the positive direction of the x-axis

$a^x \to a \times a^x \to a \times a^{2x} \to a \times a^{2(x-1)}$

$= a^1 \times a^{2x-2}$

$= a^{2x-1} \neq g(x)$

$\therefore$ C

D. dilation by a factor of a^3 from the x-axis, then translation by 1 unit in the positive direction of the x-axis, then dilation by a factor of $\frac{1}{2}$ from the y-axis

$a^x \to a^3 \times a^x \to a^3 \times a^{x-1} \to a^3 \times a^{2x-1}$

$= a^{2x-1+3}$

$= a^{2x+2} = g(x)$

End of Section A

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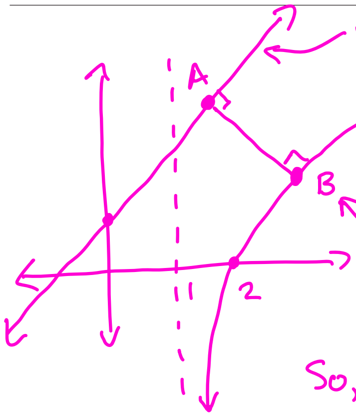
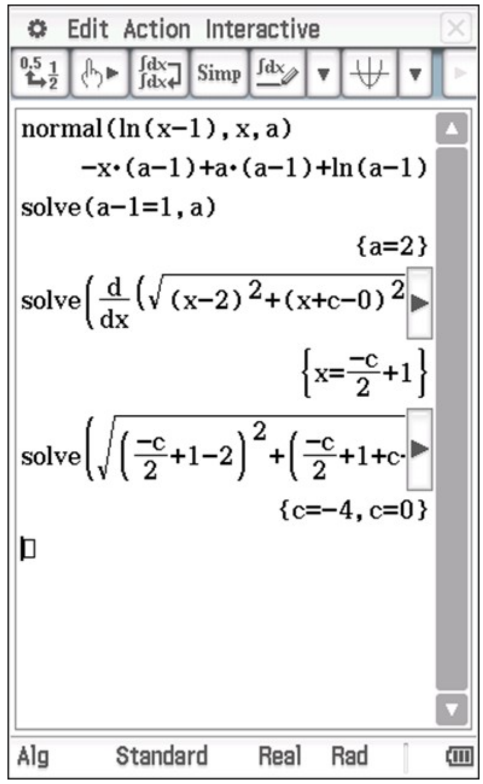


$m=1$

The minimum length of AB occurs when it is perpendicular to the line and curve.

So, as the line has gradient $=1$, find when the normal to the curve has gradient $=-1$

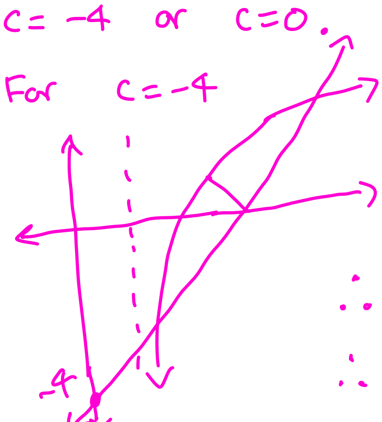


The derivative of Distance AB $=0$ when $x=-\frac{c}{2}+1$

If the Distance AB at $x=-\frac{c}{2}+1$ is equated to $\sqrt{2}$ then

$c=-4$ or $c=0$.

For $c=-4$



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$\therefore$ $c=0$

$\therefore$ D

distance is maximised (not minimised)

So $c \neq -4$

plus, $c=-4$ is not an option given

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Section B

Instructions

- Answer **all** questions in the spaces provided.
- Write your responses in English.
- In questions where a numerical answer is required, an **exact** value must be given unless otherwise specified.
- In questions where more than one mark is available, appropriate working **must** be shown.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (13 marks)

Let $g : R \to R$ be defined by $g(x) = 4x^3 - 3x^4$.

* answer part (a) while doing the graphing in part (b)

a. Find the coordinates of both stationary points of g. 2 marks

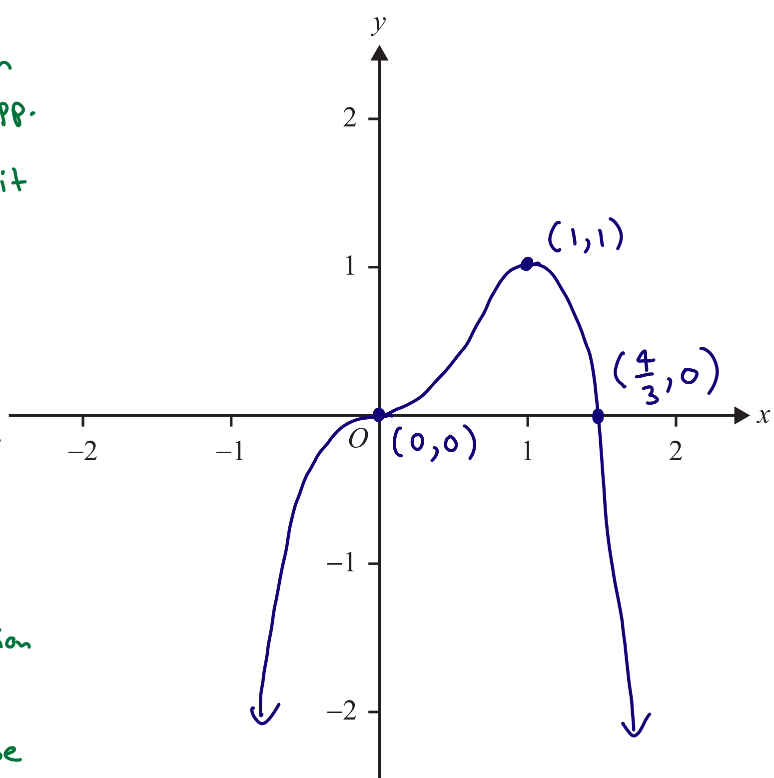
(0,0) and (1,1)

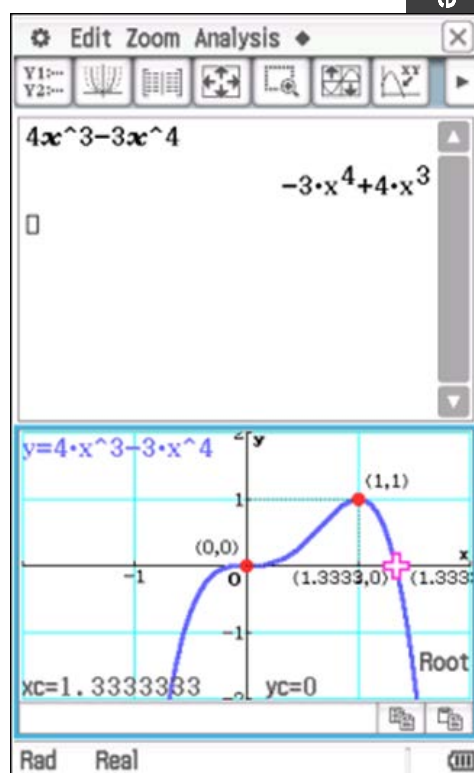
b. Sketch the graph of $y = g(x)$ on the axes below, labelling the stationary points and axial intercepts with their coordinates. 2 marks

- enter the expression in the Main app.
- drag + drop it into a G+T window.
- adjust the view window to match the axes given
- Use
  - Analysis
    - G. Solve
      - Inflection
      - Max
      - Root

* in this case exact values were obtained

if they were not:
- return to Main window
- use fMax etc





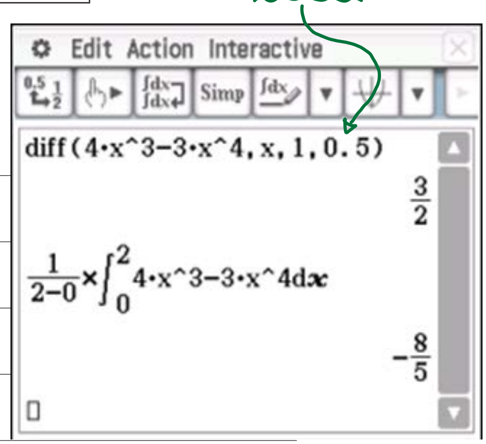
choose $x<0$ choose $0<x<1$

c. Complete the following gradient table with appropriate values of x and $g'(x)$ to show that g has a stationary point of inflection. 2 marks

x	-1	0	$\frac{1}{2}$
$g'(x)$	24	0	$\frac{3}{2}$

Calculate "deriv. at value" • Interactive • Calculation • Diff

Once one calculation is done, edit the x-value for the next



d. Find the average value of g between $x = 0$ and $x = 2$.

$$\frac{1}{2-0}\int_0^2 g(x)\,dx$$

$$= -\frac{8}{5}$$

* 2 marks ⇒ working required

e. Let h be the result after applying a sequence of transformations to g, such that h has a stationary point of inflection at $(1, 0)$ and a local maximum at $(-1, 1)$.

Write down a possible sequence of three transformations to map from g to h. 3 marks

1. dilation from the y-axis by a factor of 2
2. reflection in the y-axis
3. translation in the positive x-direction by 1 unit

many possible answers

f. Let $X \sim \text{Bi}(4, p)$ be a binomial random variable.

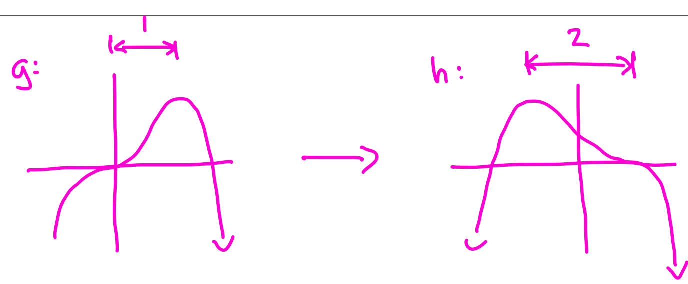
Show that $\Pr(X \geq 3) = g(p)$ for all $p \in [0, 1]$. 2 marks

$$\Pr(X \geq 3) = \binom{4}{3}p^3(1-p) + \binom{4}{4}p^4$$

$$= 4p^3 - 4p^4 + p^4$$

$$= 4p^3 - 3p^4$$

$$= g(p)$$



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Question 2 (14 marks)

Let

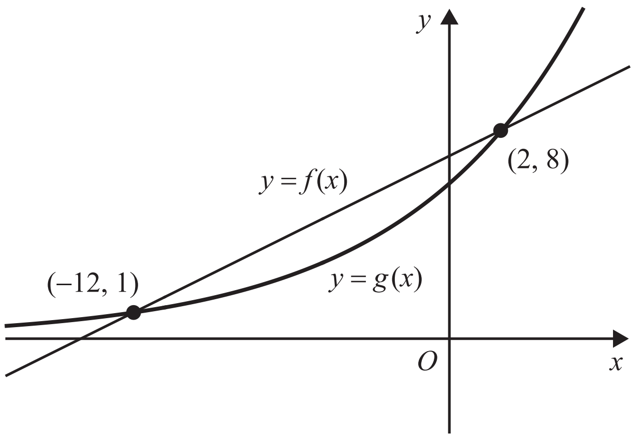
$$f : R \to R,\ f(x) = \frac{x}{2} + 7$$

and

$$g : R \to R,\ g(x) = Ae^{kx}$$

where $A, k \in R$.

The graphs of $y = f(x)$ and $y = g(x)$ intersect at the points $(-12, 1)$ and $(2, 8)$, as shown below.



a. Write down two simultaneous equations in terms of A and k.

Solve them, using algebra, to show that $A = 2^{\frac{18}{7}}$ and $k = \frac{3}{14}\log_e(2)$. 3 marks

$\frac{x}{2} + 7 = Ae^{kx}$ at

$(-12, 1) \Rightarrow Ae^{-12k} = \frac{-12}{2} + 7 = 1$

$(2, 8) \Rightarrow Ae^{2k} = \frac{2}{2} + 7 = 8$

$\therefore \frac{Ae^{2k}}{Ae^{-12k}} = \frac{8}{1}$

$\Rightarrow e^{14k} = 8$

$\Rightarrow 14k = \ln 8 = 3\ln 2$

$\Rightarrow k = \frac{3}{14}\ln 2$

$\therefore Ae^{\frac{6}{14}\ln 2} = 8$

$\Rightarrow A \times 2^{6/14} = 2^3$

$\Rightarrow A = \frac{2^3}{2^{6/14}} = 2^{\frac{42}{14} - \frac{6}{14}} = 2^{\frac{36}{14}} = 2^{\frac{18}{7}}$

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b. Find the value of b, where $b \in R$, such that $g(x)$ can be expressed in the form $g(x) = A \times 2^{bx}$. 1 mark

$e^{\frac{3}{14}\ln 2}$

$= e^{\ln 2^{3/14}}$

$= 2^{3/14}$ $\therefore b = \frac{3}{14}$

or

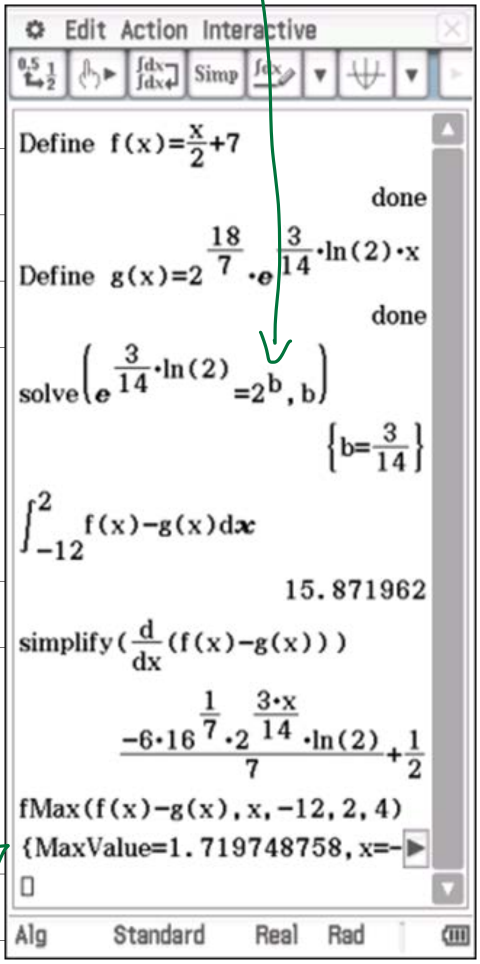
could solve $e^k = 2^b$ for b

c. Use a definite integral to evaluate the area bounded by the graphs of $y = f(x)$ and $y = g(x)$, where $x \in [-12, 2]$.

Give the area correct to two decimal places.

$\int_{-12}^{2} f(x) - g(x)\, dx$

$= 15.87$



d. Let $h(x) = f(x) - g(x)$.

i. Write down an expression for the derivative of $h(x)$.

$\frac{1}{2} - 2^{\frac{18}{7}} \times e^{\frac{3}{14}\ln 2x} \times \frac{3}{14}\ln 2$

or

ii. Find the maximum value of $h(x)$, where $x \in [-12, 2]$.

Give your answer correct to two decimal places.

1.72

or

Solve $h'(x) = 0$

(+ eval. $h(x)$ at that solution)

e. Let g^{-1} be the inverse of g.

Find the points where the graph of $y = g^{-1}(x)$ intersects with the graph of $y = 2(x - 7)$. 2 marks

$2(x-7)$ is $f^{-1}(x)$

f^{-1} meets g^{-1} at the "inverse pts" to where f meets g

$\therefore$ meets $g^{-1}(x)$ at $(1, -12)$ and $(8, 2)$

points (x, y)

$(-12, 1)$ & $(2, 8)$

"invert" to (y, x)

or

- Use CAS to find $g^{-1}(x)$
- Use CAS to solve

$g^{-1}(x) = 2(x-7)$

Question 2 continues on the next page.

f. Let F be an anti-derivative of f that passes through $(0, c)$, where $c \in R$.

i. Show that it is **not** possible for the graph of $y = F(x)$ to pass through both $(-12, 1)$ and $(2, 8)$. 2 marks

$F(x) = \int \frac{x}{2} + 7 \, dx$

$= \frac{x^2}{4} + 7x + c$

If $F(x)$ passes through $(-12, 1)$ then

$\frac{144}{4} - 84 + c = 1 \Rightarrow c = 49$

For this value of c *

$F(2) = \frac{4}{4} + 14 + 49 = 64$

$\therefore$ $F(x)$ passes through $(2, 64)$ not $(2, 8)$

* alternatively, you could use $(2,8)$ and find $c = -7$, showing that $F(x)$ can't satisfy both $(-12,1)$ and $(2,8)$ (two different c-values)

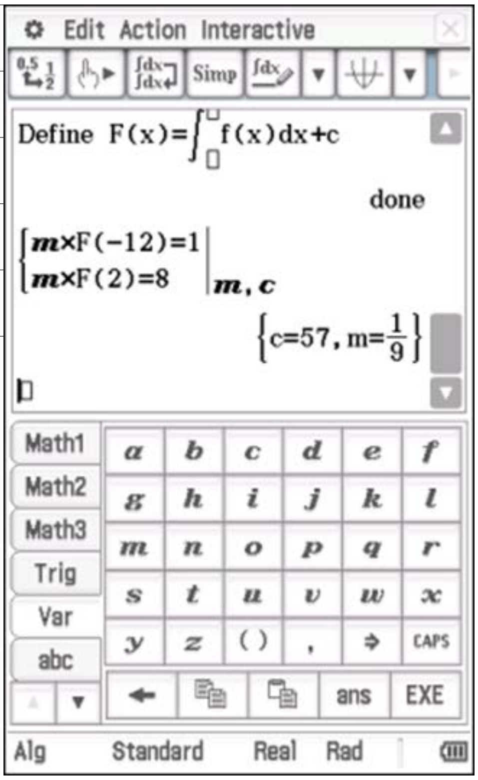
ii. The graph of $y = F(x)$ can be dilated by a factor of m from the x-axis such that its image passes through both $(-12, 1)$ and $(2, 8)$.

Find the values of m and c. 2 marks

$$\begin{cases} m \times F(-12) = 1 \\ m \times F(2) = 8 \end{cases}$$

$\therefore m = \frac{1}{9}, \ c = 57$

- Define $f(x)$ (could also use $\frac{x^2}{4} + 7x + c$)
- Use the Sim. Eq. template on Math 1 keyboard.



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Examination continues on the next page.

Question 3 (14 marks)

The time taken for a driver to travel to work each day, in minutes, is modelled by a continuous random variable T with probability density function

$$f(t) = \begin{cases} \dfrac{1}{1215000}(t-29)(59-t)^3 & 29 \le t \le 59 \\ 0 & \text{otherwise} \end{cases}$$

a. **i.** Find the mean time taken, in minutes, for the driver to travel to work each day. 1 mark

39 mins

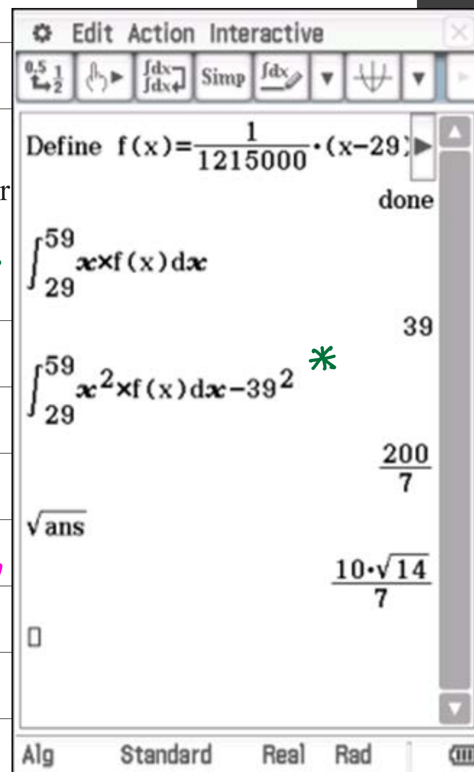
ii. Find the standard deviation of the time taken, in minutes, for the driver to travel to work each day.

$\sigma^2 = \int_{29}^{59} t^2 \times f(t)\, dt - 39^2$

* working needed *

$\sigma = \dfrac{10\sqrt{14}}{7}$

don't forget to square root!



* can do this by editing the previous calculation

b. The driver allows k minutes to travel to work each day. If the journey takes longer than k minutes, the driver will be late. Whether the driver is late on a particular day is independent of whether they are late on any other day.

i. If $k = 47$, write a definite integral to show that the probability of the driver being late is 0.08704 — 1 mark

$$\int_{47}^{59} f(t)\,dt = \frac{272}{3125} = 0.08704$$

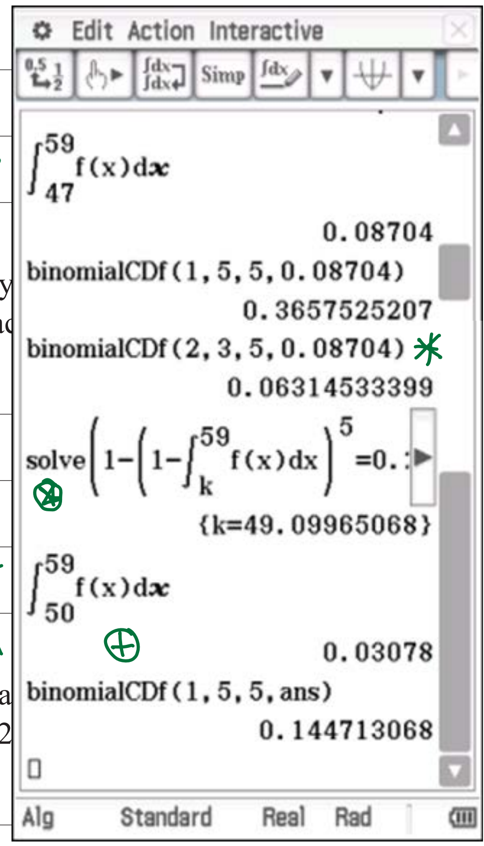
ii. If $k = 47$, find the probability that the driver will be late on at least one day in a five-day working week.

Give your answer correct to four decimal places. — 2 marks

$Y \sim Bi(5, 0.08704)$

$\Pr(Y \geq 1) = 0.3658$

watch your rounding



iii. For $k = 47$, let $\hat{P}$ be the proportion of days the driver is late in any working week. Find $\Pr(0.4 \leq \hat{P} \leq 0.6)$ correct to four decimal plac

$\Pr(0.4 \leq \hat{p} \leq 0.6) = \Pr(2 \leq Y \leq 3)$

$= 0.0631$ * can be done by editing the previous calculation

iv. Find the **integer** k such that the probability, correct to one decima driver being late at least once in any five-day working week is 0.2

$\Pr(Y \geq 1) = 0.2$

$\Rightarrow 1 - \Pr(Y = 0) = 0.2$

$\Rightarrow 1 - (\text{"on time"})^5 = 0.2$

$\Rightarrow 1 - (1 - \Pr(\text{"late"}))^5 = 0.2$

$1 - \left(1 - \int_k^{59} f(x)\,dx\right)^5 = 0.2$

$\therefore k = 49$

or

k	Prob
48	0.28106
49	0.20675
50	0.14471

$\therefore k = 49$

solve this equation "numerically" - it's faster and an answer to the nearest integer is sufficient

⊕ Use "ans" like this to speed up T&E

Question 3 continues on the next page.

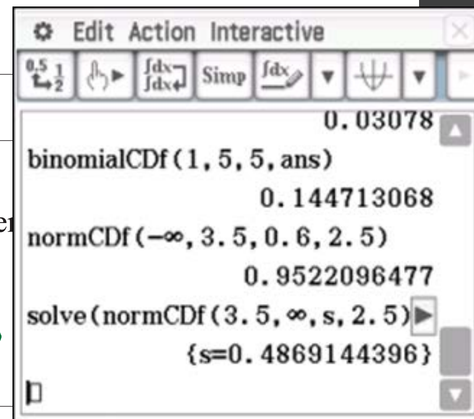
c. At a given traffic light, the wait time is modelled by a normal distribution with a mean of 2.5 minutes and a standard deviation of σ minutes.

i. If $\sigma = 0.6$, find the probability that the wait time will be less than 3.5 minutes.

Give your answer correct to two decimal places. 1 mark

0.95

Found in Main via
• Interactive • Distribution
• Continuous • norm CDF



ii. Find the value of σ such that there is a 2% chance of a wait time longer than 3.5 minutes.

Give your answer correct to two decimal places.

0.49

• Edit a norm CDF command (like the previous)
• include a variable for σ and solve.

d. The driver passes through three traffic lights (A, B and C) on their journey to work. The probability of each traffic light being red is shown in the table below.

Traffic light	A	B	C
Probability that the traffic light is red	0.2	0.3	0.1

Let Y be the random variable representing the number of traffic lights that are red on the driver's journey to work. Assume that each traffic light being red is independent of any other traffic light being red.

Complete the following table for the probability distribution of Y. 2 marks

y	0	1	2	3
$\Pr(Y = y)$	0.504	0.398	0.092	0.006

$0.8 \times 0.7 \times 0.9$

$0.2 \times 0.3 \times 0.1$

$0.2 \times 0.7 \times 0.9 + 0.8 \times 0.3 \times 0.9 + 0.8 \times 0.7 \times 0.1$

$0.2 \times 0.3 \times 0.9 + 0.2 \times 0.7 \times 0.1 + 0.8 \times 0.3 \times 0.1$

using CP with "swipe-drag-drop" can speed up these repetitive calculations

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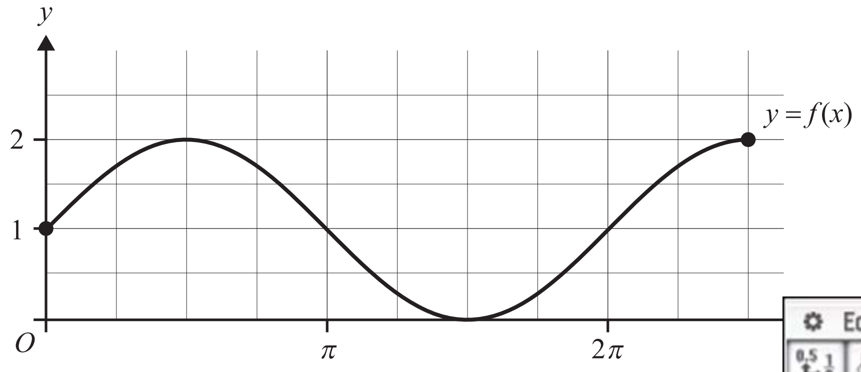
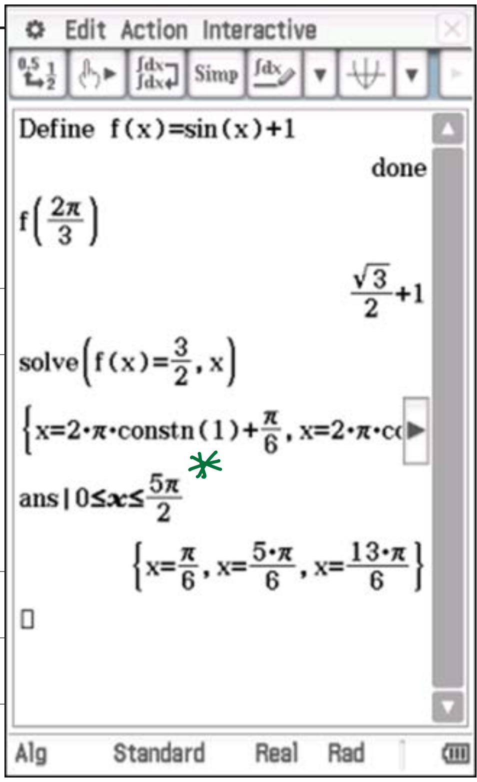
Question 4 (19 marks)

Consider the function $f: \left[0, \frac{5\pi}{2}\right] \to R,\ f(x) = \sin(x) + 1$.

The graph of $y = f(x)$ is shown below.





a. Evaluate $f\left(\frac{2\pi}{3}\right)$.

$\frac{\sqrt{3}}{2} + 1$

b. Find the exact values of x for which $f(x) = \frac{3}{2}$.

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$

* solve $f(x) = \frac{3}{2}$ exactly (no bounds) then set the domain on the answer to get the three solutions.

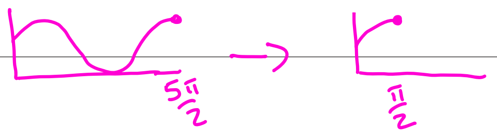
c. There exist real numbers a and k in the interval $\left(0, \frac{5\pi}{2}\right)$, such that $f(x+k) = f(x)$ for all $x \in [0, a]$.

definition of function with period of k

Find the value of k and the largest possible value of a. 2 marks

$k = 2\pi$ (period of $f(x)$)

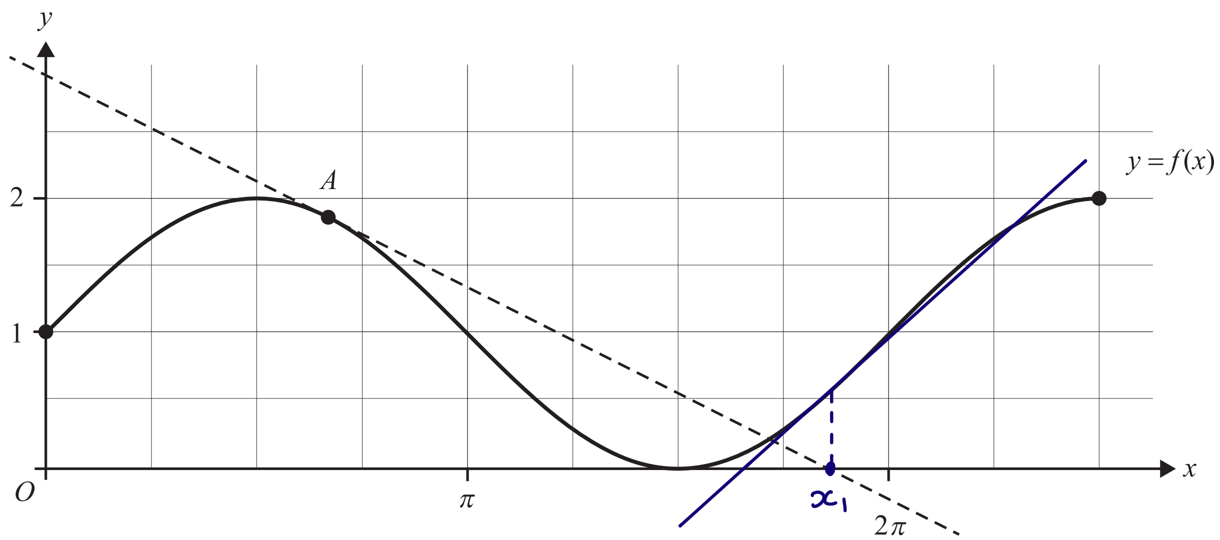
$a = \frac{\pi}{2}$



F(x) is translated left by $k = 2\pi$ units

Question 4 continues on the next page.

d. Consider the tangent to the graph of $y = f(x)$ at the point A where $x = \frac{2\pi}{3}$, as shown on the axes below.

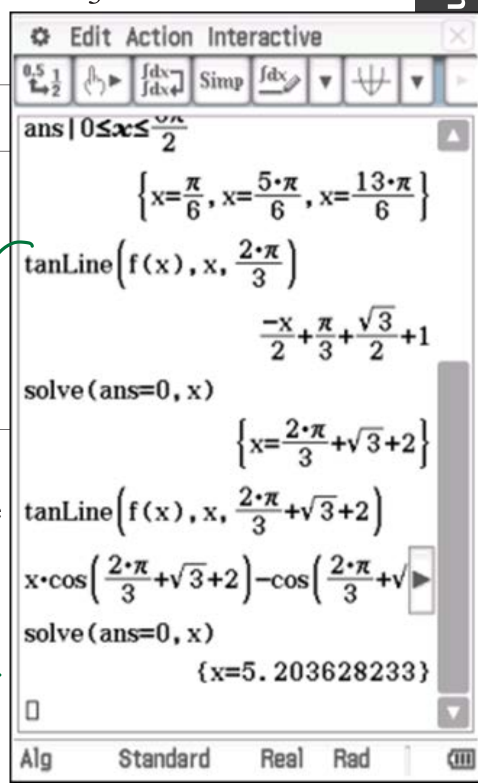


Find the equation of the tangent to the graph of $y = f(x)$ at the point where $x = \frac{2\pi}{3}$. 1 mark

$y = \frac{x}{2} + \frac{\pi}{3} + \frac{\sqrt{3}}{2} + 1$

use
• Interactive • Calculation
• line • tanLine

don't forget the "y="



e. Apply two iterations of Newton's method to f with $x_0 = \frac{2\pi}{3}$.

i. Write down x_2, correct to one decimal place.

$x_2 = 5.2$

ii. On the axes in **part d**, draw the tangent to the graph of $y = f(x)$ at the point where $x = x_1$.

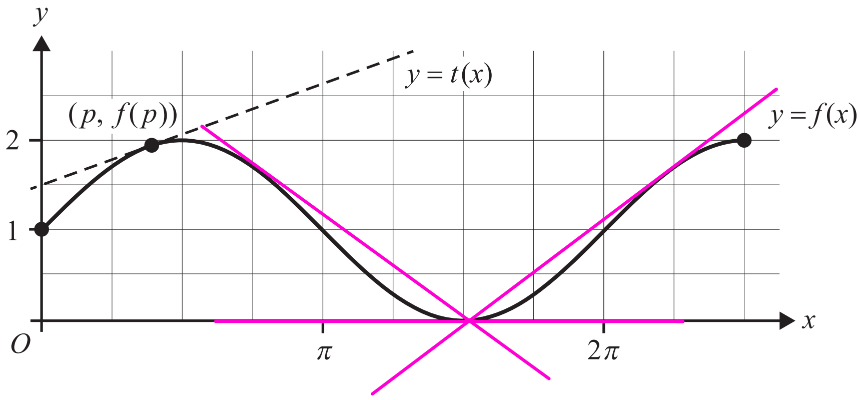
(*Answer on the graph in* ***part d***.)

* don't miss this mark

* use a ruler

Some swipe-drag-drop makes light work of this iterative process.

f. Now consider the line $y = t(x)$, which is the tangent to the graph of $y = f(x)$ at the point $(p, f(p))$, where $p \in \left(0, \frac{5\pi}{2}\right)$.



i. Show that $t(x) = \cos(p)(x - p) + \sin(p) + 1$. 2 marks

$f'(x) = \cos x$

$\therefore m = f'(p) = \cos p$

$\therefore \dfrac{y - (\sin p + 1)}{x - p} = \cos p$

$\therefore y = \cos p\,(x - p) + \sin p + 1$

* other methods for finding the equation of a tangent can be used.

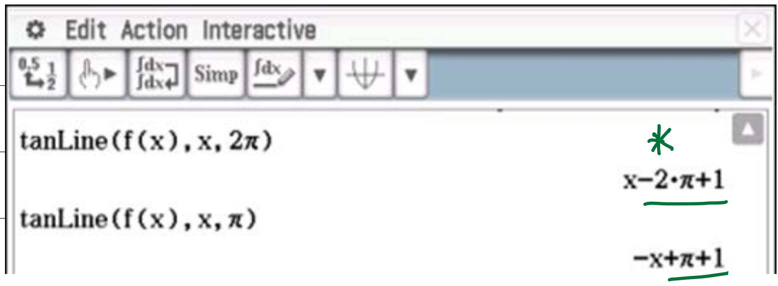
ii. Determine the minimum and maximum possible values for the y-intercept of $y = t(x)$, for $p \in \left(0, \frac{5\pi}{2}\right)$. 2 marks

min. y-int is at infl. pt at $x = 2\pi$

$\therefore$ min y-int is $-2\pi + 1$ *

max. y-int is at infl. pt at $x = \pi$

$\therefore$ max y-int is $\pi + 1$ ⊕



*

⊕

iii. Determine the values of p for which $y = t(x)$ has a unique x-intercept that is equal to the x-intercept of $y = f(x)$.

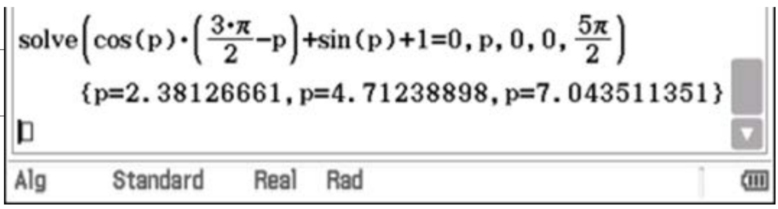
Give your answers correct to two decimal places. $\therefore$ solve numerically 2 marks

x-int of $f(x)$ is $\frac{3\pi}{2}$

$\cos(p) \times \left(\frac{3\pi}{2} - p\right) + \sin(p) = 0$

for $p = 2.38$ and $p = 7.04$

($p \neq 4.712 = \frac{3\pi}{2}$ as a unique x-int is required)



Question 4 continues on the next page.

g. Let $g : \left[0, \frac{5\pi}{2}\right] \to R,\ g(x) = ax^3 + bx^2 + cx + d$ be a polynomial function, where $a, b, c, d \in R$.

Suppose $g(0) = f(0)$ and $g'(0) = f'(0)$.

i. Show that $c = 1$ and $d = 1$. 2 marks

$f(0) = 1 = g(0) \quad \therefore\ 0 + 0 + 0 + d = 1 \quad \therefore\ d = 1$

$f'(0) = 1 = g'(0) \quad \therefore\ 0 + 0 + c = 1 \quad \therefore\ c = 1$

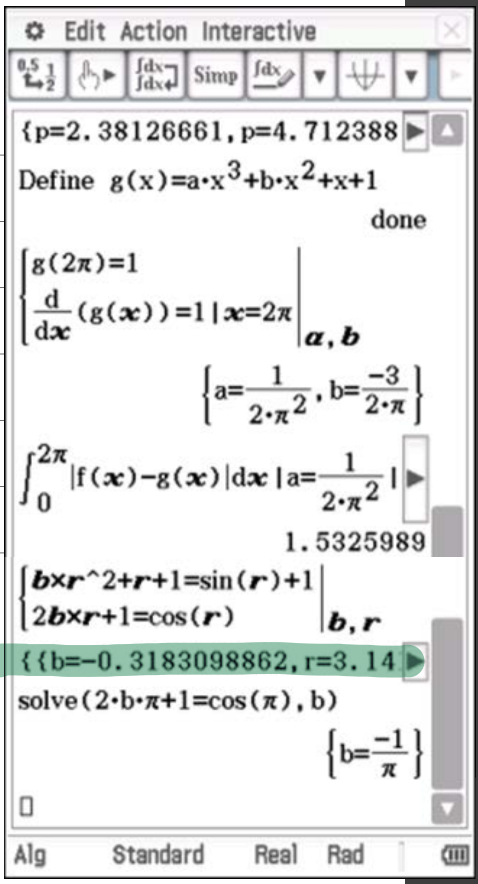
ii. If $g(2\pi) = f(2\pi)$ and $g'(2\pi) = f'(2\pi)$, determine the area bounded by the graphs of $y = f(x)$ and $y = g(x)$, for $x \in [0, 2\pi]$.

Give your answer correct to two decimal places.

$\begin{cases} g(2\pi) = 1 \\ g'(x) = 1 \mid x = 2\pi \end{cases}$

$\therefore\ a = \frac{1}{2\pi^2}, \quad b = \frac{-3}{2\pi}$

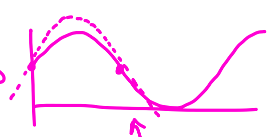
$\therefore \int_0^{2\pi} |f(x) - g(x)|\, dx = 1.53$



iii. Let $a = 0$, $c = 1$, $d = 1$.

tangent at $(0, 1)$ from part (i)

Find b and r, such that $g(r) = f(r)$ and $g'(r) = f'(r)$, where $b \in R$ and $r \in \left(0, \frac{5\pi}{2}\right)$.



For $g(x) = bx^2 + x + 1$

$\begin{cases} br^2 + r + 1 = \sin r + 1 \\ 2br + 1 = \cos r \end{cases}$

$r = \pi$ **

approx sol. suggests $r = \pi$

also suggested by symmetry of sine and of parabolas

$2 \times b \times \pi + 1 = \cos \pi$

$\therefore\ b = \frac{-2}{2\pi} = \frac{-1}{\pi}$

End of examination questions

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