Solutions with CASIO fx-8200AU calculator help for the NSW HSC Mathematics (Advanced) examination 2025.

Make sure your calculator settings are on Math I/MathO to allow for exact surds or fraction answers.
You can easily convert to decimals using the FORMAT button or SHIFT EXE.

Suggested solutions in blue.

Extra notes in pink.

CASIO help in green.

2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 2–8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9–40)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

1 The probability distribution table for a discrete random variable X is shown.

x	$P(X = x)$
1	0.4
2	0.2
3	0.4

What is the value of $P(X = 3)$?

Sum of all the probabilities is 1.

$$P(X=3) = 1 - 0.4 - 0.2$$
$$= 0.4$$

A. 0.2

[✓ B.] 0.4

C. 1.2

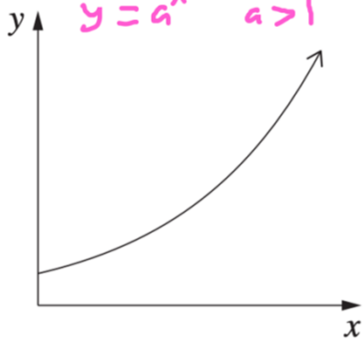
D. 2.0

Probabilities not in range: $0 \le P(x) \le 1$

You can check by producing a table of values on the calculator. Note: at $x=0$, $f(x) = 4^0 = 1$, At $x=1$, $f(x) = 4$. This eliminates all answers, except A.

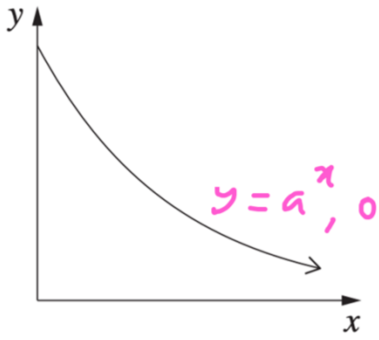
2 Which graph could represent $y = 4^x$? Increasing exponential function.

[✓ A.]



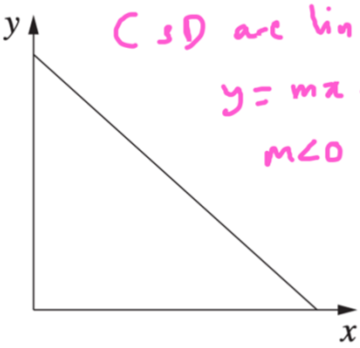
$y = a^x$ $a > 1$

B.



$y = a^x$, $0 < a < 1$

C.

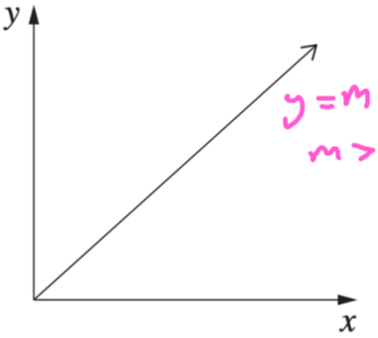


C & D are linear.

$y = mx + c$

$m < 0$

D.

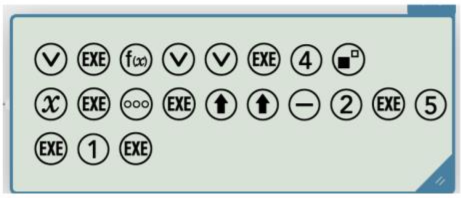


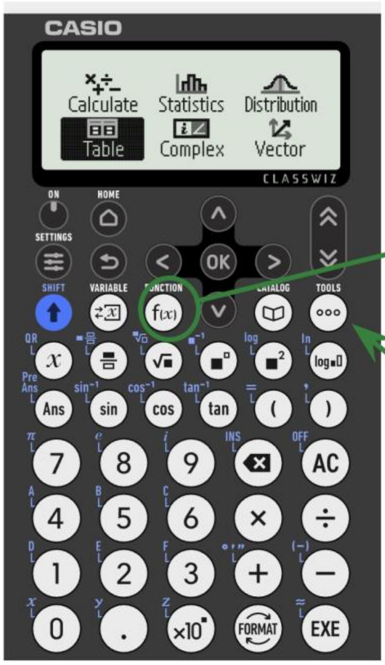
$y = mx + c$

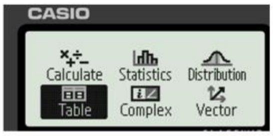
$m > 0$

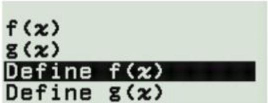
You can check by producing a table of values on the calculator. Note: at $x = 0$, $f(x) = 4^0 = 1$, At $x = 1$, $f(x) = 4$. This eliminates all answers, except A.

KEY LOG:









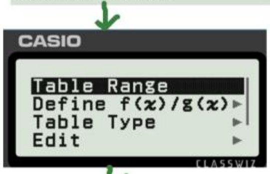
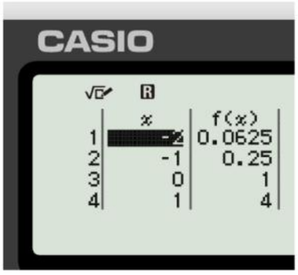


Table Range
Start:-2
End :5
Step :1

Table mode shows $f(x) = 4^x$



	x	f(x)
1	-2	0.0625
2	-1	0.25
3	0	1
4	1	4

As x increases, $y = f(x)$ increases.

possible x-values

3 What is the domain of the function $y=\sqrt{6-x^2}$?

A. $(0, \sqrt{6})$

B. $[0, \sqrt{6}]$

C. $(-\sqrt{6}, \sqrt{6})$

[✓ D.] $[-\sqrt{6}, \sqrt{6}]$

We require $6-x^2 \geq 0$

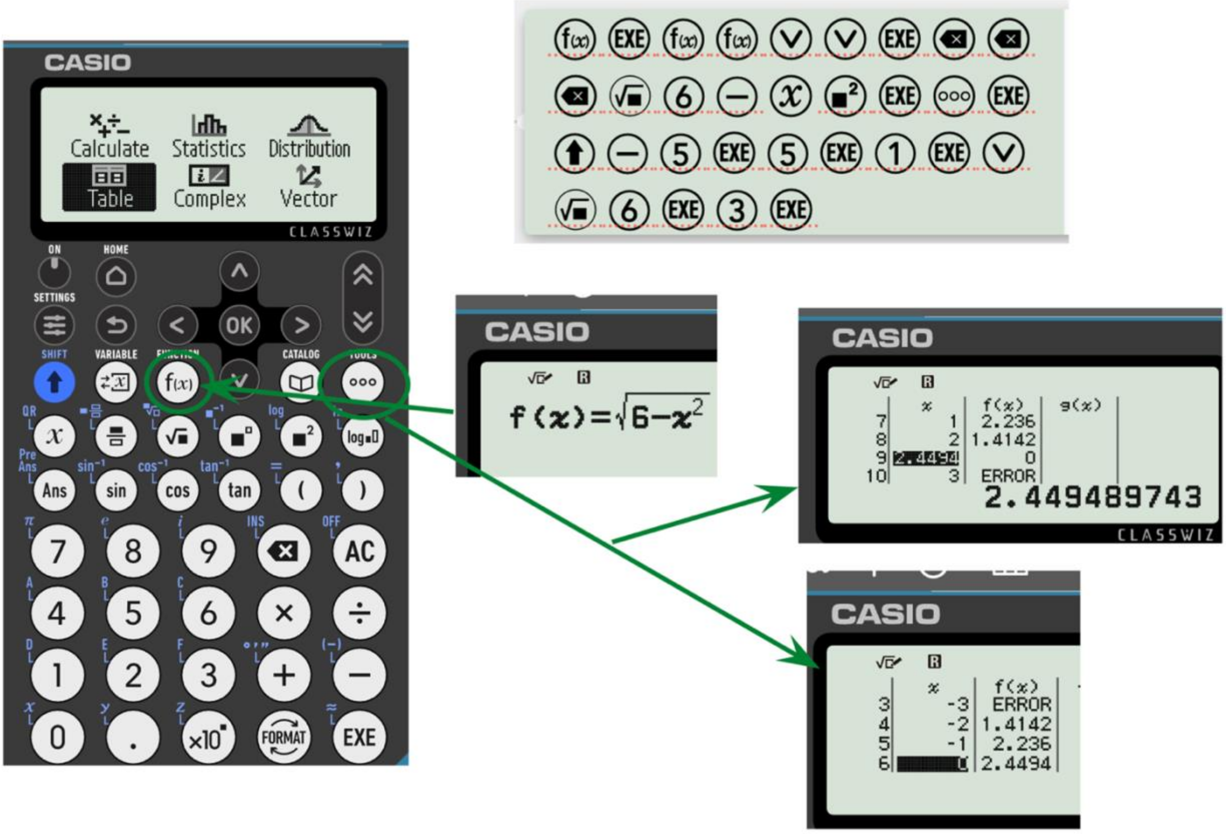
$x^2 \leq 6$

$-\sqrt{6} \leq x \leq \sqrt{6}$

Check on TABLE mode of the calculator.

For $x=-1$, $y=f(x)=2.236\ldots$, eliminating A & B.

For $x=\sqrt{6}=2.449\ldots$, eliminating C.



```
f(x)=√(6−x²)
```

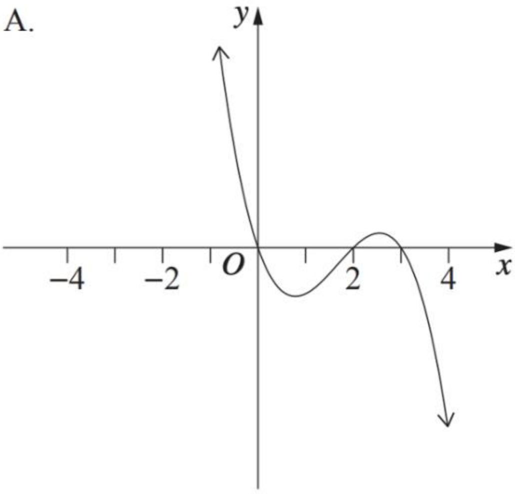
```
     x   f(x)    g(x)
 7    1  2.236
 8    2  1.4142
 9 2.4494     0
10    3  ERROR
     2.449489743
```

```
     x   f(x)
 3   -3  ERROR
 4   -2  1.4142
 5   -1  2.236
 6       2.4494
```

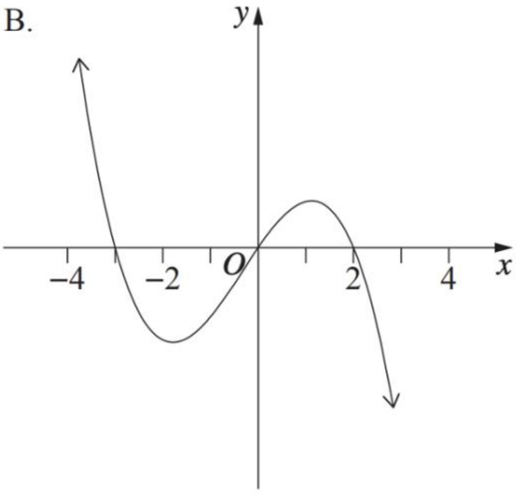
x-intercepts at $x = 0,\ 2,\ 3$. Leading term is $5x^3$ $\therefore$ C

4 Which of the following best represents the graph of $y = -5x(x-2)(3-x)$?

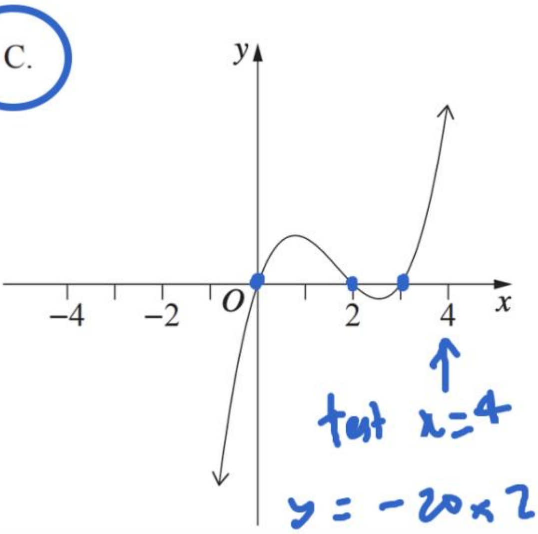
A.



B.



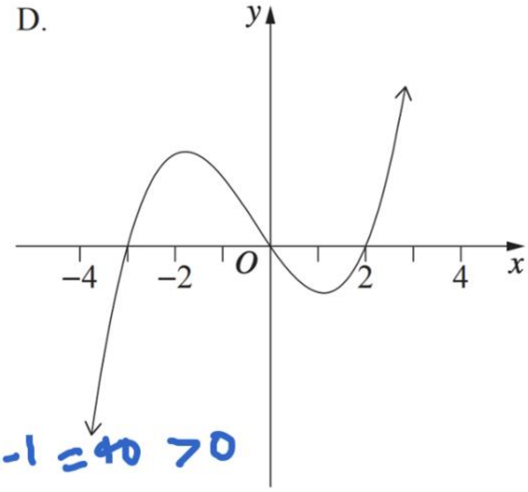
[✓ C.]

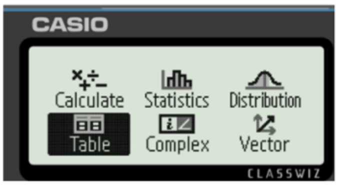


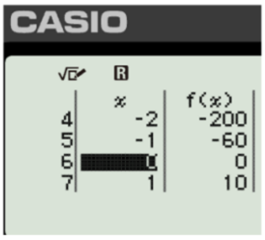
test $x=4$

$y = -20 \times 2 \times -1 = 40 > 0$

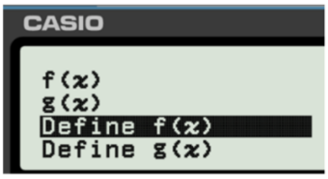
D.

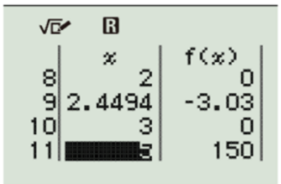






	x	f(x)
4	-2	-200
5	-1	-60
6	0	0
7	1	10





	x	f(x)
8	2	0
9	2.4494	-3.03
10	3	0
11	4	150

Use TABLE mode.

Let $f(x) = -5x(x-2)(3-x)$

You can easily find the x-intercepts at $x = 0, 2, 3$.

Also at $x = -1$, $y = -60$

5 What is $\int \frac{1}{\sqrt{x+5}}\,dx$? $= \int (x+5)^{-\frac{1}{2}}\,dx$

$$= \frac{(x+5)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2\sqrt{x+5} + C$$

A. $\frac{1}{2}\sqrt{x+5} + C$

[✓ B.] $2\sqrt{x+5} + C$

C. $-\frac{1}{2}\sqrt{x+5} + C$

D. $-2\sqrt{x+5} + C$

Check:

$$\frac{d}{dx} 2\sqrt{x+5} = \frac{d}{dx} 2(x+5)^{\frac{1}{2}}$$

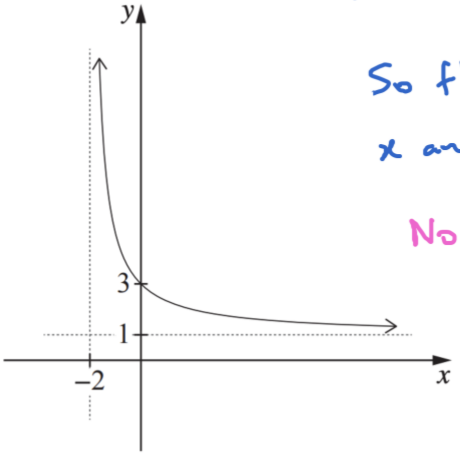
$$= 2 \times \frac{1}{2} \times (x+5)^{-\frac{1}{2}} = \frac{1}{\sqrt{x+5}}$$

$f(-x)$ is reflection of $f(x)$ in y-axis

$-f(-x)$ is reflection of $f(-x)$ in x-axis.

So flip original graph in both the x and y-axes. (Order not important).

6 The graph of $y = f(x)$ is shown.



Note $f(0)=3$

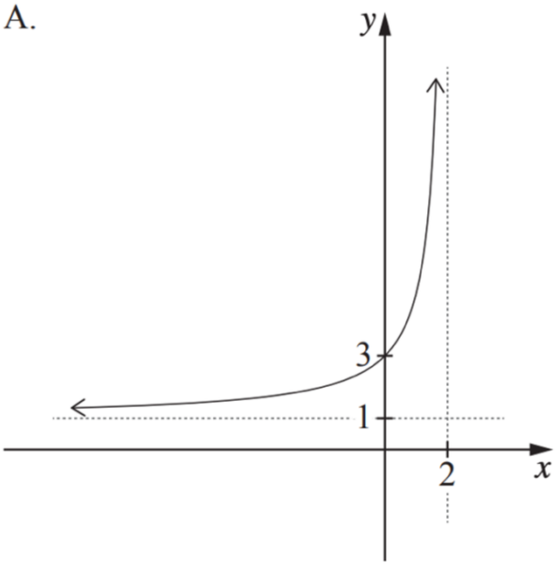
$\therefore -f(-0) = -f(0) = -3$

So graph of $-f(-x)$ will have $y=-3$ at $x=0$. Eliminating A+B.

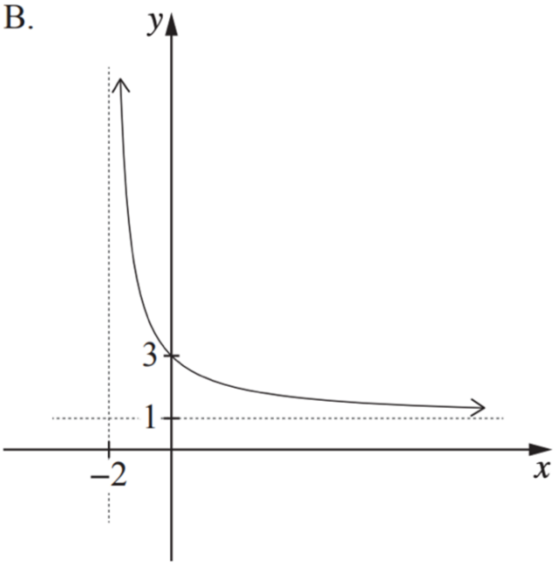
Which of the following is the graph of $y = -f(-x)$?

Answer is C.

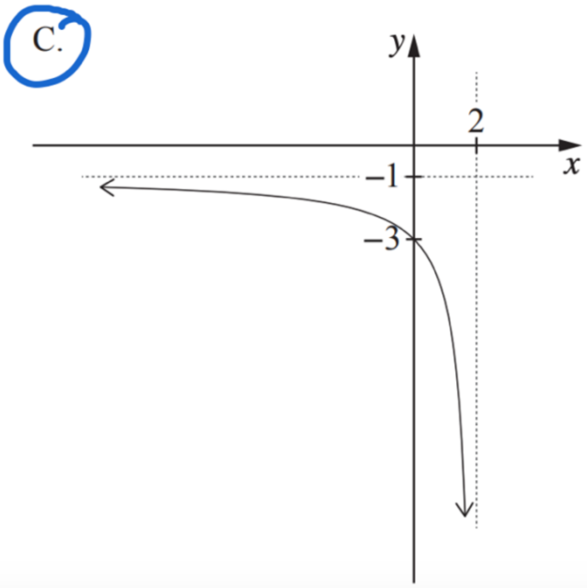
A.



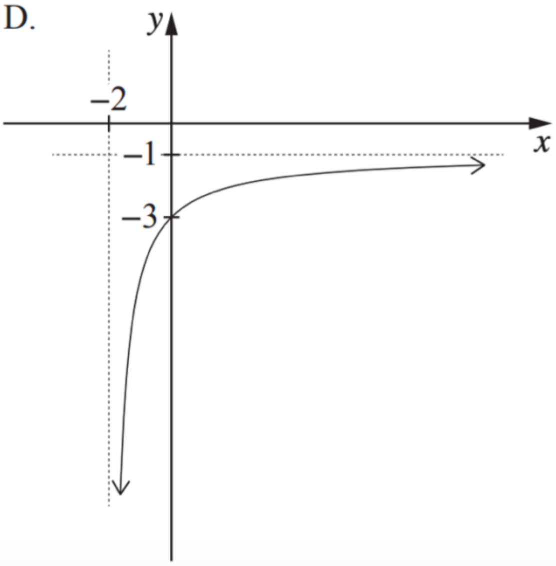
B.



[✓ C.]



D.



7 A ten-sided die has faces numbered 1 to 10.

The die is constructed so that the probability of obtaining the number 1 is greater than the probability of obtaining any of the other numbers. The numbers 2 to 10 are equally likely to occur.

When the die is rolled 153 times, a 1 is obtained 72 times. $P(X=1)=\frac{72}{153}=\frac{8}{17}$

By using the relative frequency of rolling a 1, which of the following is the best estimate for the probability of rolling a 10? Let $P(X=10)=p$

[✓ A.] $\frac{1}{17}$

B. $\frac{1}{11}$

C. $\frac{1}{10}$

D. $\frac{1}{9}$

Sum of probabilities = 1

$$\therefore \frac{8}{17}+9p=1$$

$$9p=1-\frac{8}{17}$$

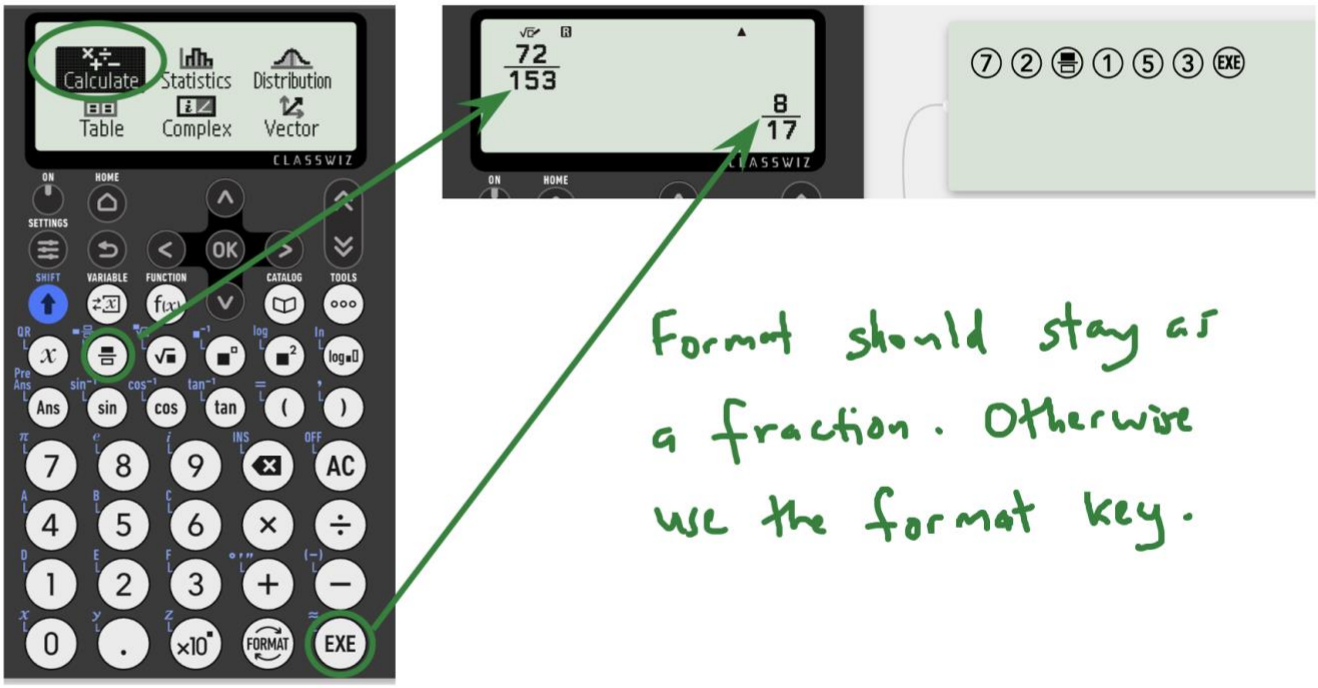
$$=\frac{9}{17}$$

$$\therefore p=\frac{1}{17}$$

Table of relative frequencies:

x	1	2	3	4	5	6	7	8	9	10
$P(X=x)$	$8/17$	p	p	p	p	p	p	p	p	p

To get equivalent fraction of $\frac{72}{153}$:



⑦ ② ⊟ ① ⑤ ③ EXE

Format should stay as a fraction. Otherwise use the format key.

8 The minimum daily temperature, in degrees, of a town each year follows a normal distribution with its mean equal to its standard deviation. The minimum daily temperature was recorded over one year. $\mu = \sigma$

$x > 0$

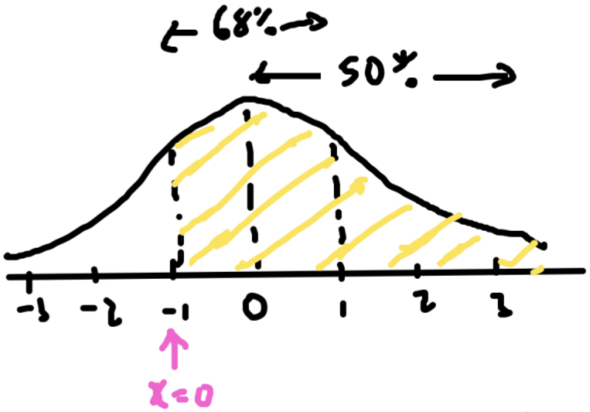
What percentage of the recorded minimum daily temperatures was above zero degrees?

A. 16%

B. 50%

C. 68%

[✓ D.] 84%



$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{0-\mu}{\mu} \quad \text{(since } \mu=\sigma\text{)}$$

$$= -1$$

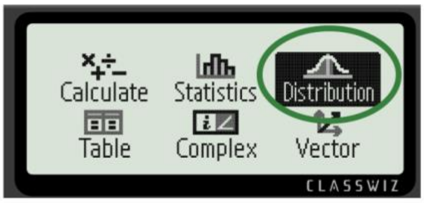
Need percentage of scores above $x = 0$ $(z = -1)$

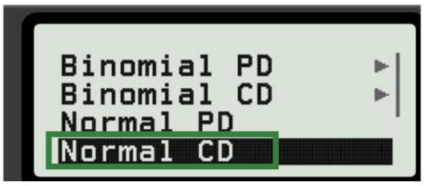
$$\text{Percentage} = \frac{68}{2} + 50$$

$$= 84\%$$

Check on calculator: set $\mu = \sigma = 1$

(will work for $\mu = \sigma = 2$, etc.)





Normal CD

Lower:0

Upper:1000

μ :1

σ :1

Execute

P=

0.8413447461

Probability = 0.841...

≈ 84%

$f'(1) > f'(1.1) > 0$

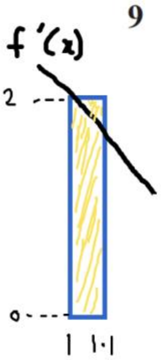
So as x increases from 1 to 1.1

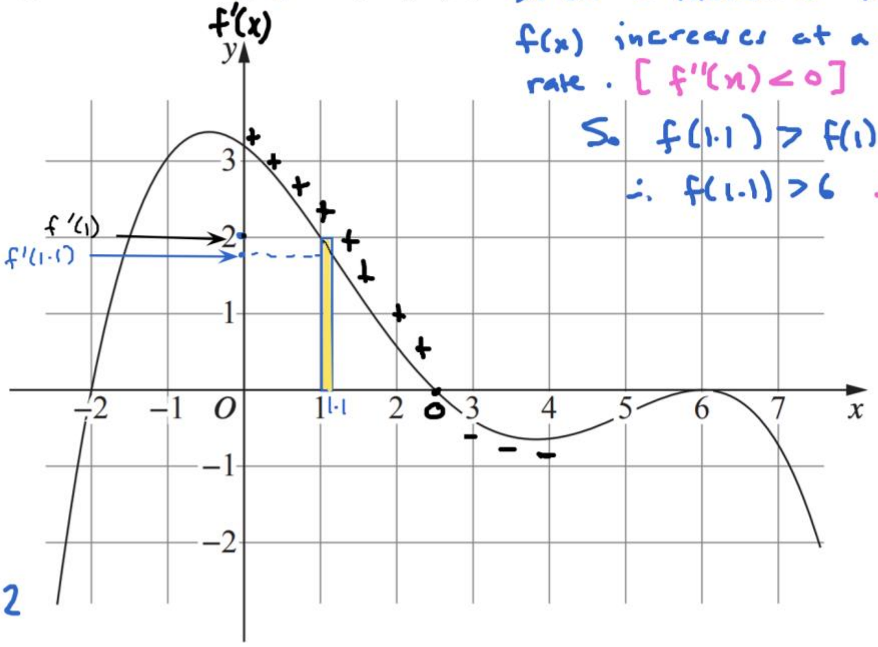
$f(x)$ increases at a decreasing rate. $[f''(x) < 0]$

So $f(1.1) > f(1) = 6$

$\therefore f(1.1) > 6$ ∴ not C or D.

9 The diagram shows the graph of $y = f'(x)$.





Area of rectangle is $0.1 \times 2 = 0.2$

So $f(x)$ has increased by less than 0.2

$\therefore 6 < f(1.1) < 6.2$

Given $f(1) = 6$, which interval includes the best estimate for $f(1.1)$?

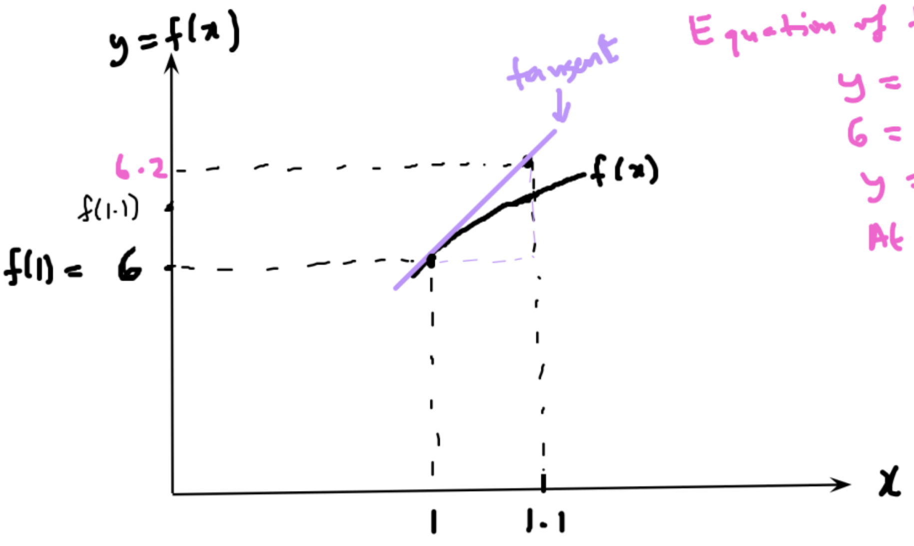
A. $[6.2, 6.4)$

[✓ B.] $[6.0, 6.2)$

C. $[5.8, 6.0)$

D. $[5.6, 5.8)$

Alternative method: Consider tangent to curve of $f(x)$.



Equation of tangent at $x = 1$ is

$y = mx + c$

$6 = 2(1) + c \Rightarrow c = 4$

$y = 2x + 4$

At $x = 1.1$, $y = 2(1.1) + 4 = 6.2$

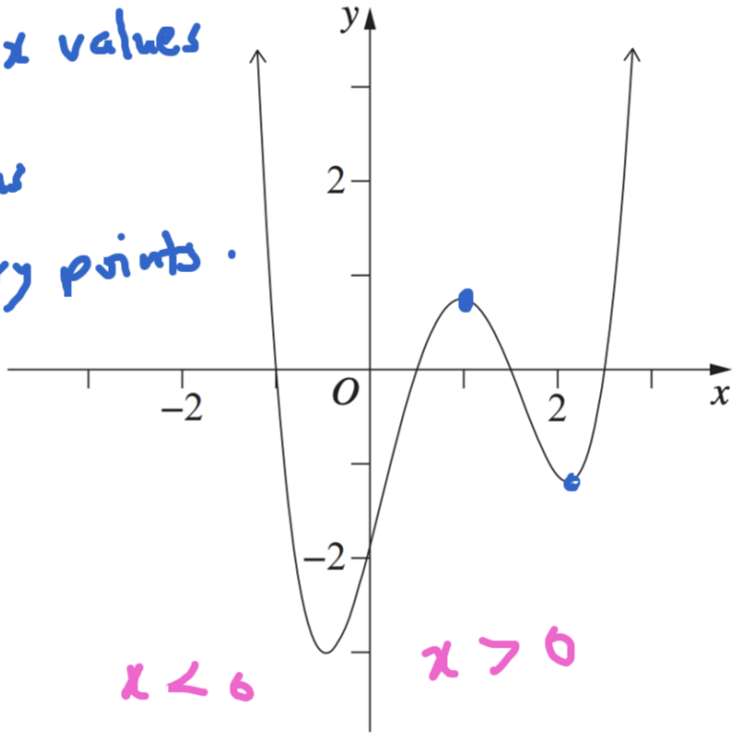
So $6 < f(1.1) < 6.2$

10 The graph of $y = f(x)$, with all its stationary points, is shown.

$e^x > 0$ for all x values

So $f(e^x)$ has

two stationary points.



How many stationary points does the graph of $y = f(e^x)$ have?

A. 0

B. 1

[✓ C.] 2

D. 3

2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Centre Number

Mathematics Advanced

Section II Answer Booklet

Student Number

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your Centre Number and Student Number at the top of this page
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response
- Your responses should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering

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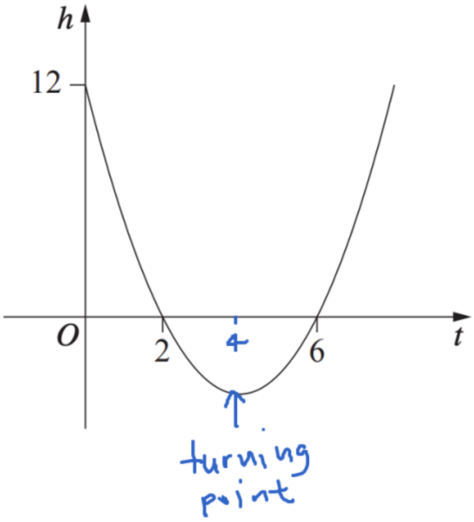
Please turn over

Question 11 (3 marks)

The graph of a quadratic function represented by the equation $h = t^2 - 8t + 12$ is shown.



Vertex or turning point is half-way between the t-axis intercepts 2 and 4.

(a) Find the values of t and h at the turning point of the graph. **2**

$t = \frac{(2+6)}{2} = 4$

$h(4) = (4)^2 - 8(4) + 12$

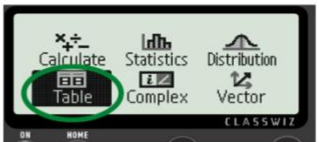
$= -4$

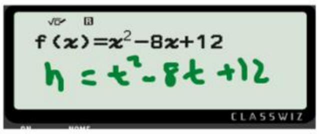
(b) The graph shows $h = 12$ when $t = 0$. $(t = 4-4 = 0)$ **1**

What is the other value of t for which $h = 12$?

By symmetry, other point is $t = 4+4 = 8$

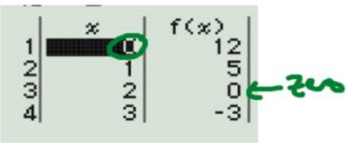
Check on calculator using TABLE mode.



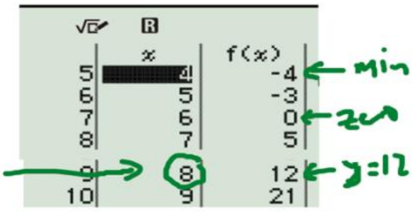


$f(x) = x^2 - 8x + 12$

$h = t^2 - 8t + 12$



	x	$f(x)$
1	0	12
2	1	5
3	2	0
4	3	-3



	x	$f(x)$
5	4	-4
6	5	-3
7	6	0
8	7	5
9	8	12
10	9	21

Answer:
$t = 8$

Question 12 (3 marks)

Find the equation of the tangent to $y = 5x^3 - \frac{2}{x^2} - 9$ at the point $(1, -6)$. **3**

$y = 5x^3 - 2x^{-2} - 9$

$y' = \frac{dy}{dx} = 15x^2 + 4x^{-3}$ ✓

Gradient of tangent at $(1, -6)$ is :

$m = \left.\frac{dy}{dx}\right|_{x=1} = 15(1)^2 + 4(1)^{-3} = 19$ ✓

Equation of tangent :

$y - y_1 = m(x - x_1)$

$y - (-6) = 19(x - 1)$

$y + 6 = 19x - 19$

$y = 19x - 25$ ✓

OR:

$y = mx + c$

$-6 = 19(1) + c$

$c = -25$

$\therefore y = 19x - 25$

Question 13 (2 marks)

Geometric sequence:

$$\begin{array}{cccc} a, & ar, & ar^2, & ar^3 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ T_1 & T_2 & T_3 & T_4 \end{array}$$

The numbers, 75, *p*, *q*, 2025, form a geometric sequence. **2**

Find the values of *p* and *q*.

$T_1 = 75 \Rightarrow a = 75$

$T_4 = 2025 \Rightarrow 75r^3 = 2025$ ✓

$r^3 = \frac{2025}{75} = 27$

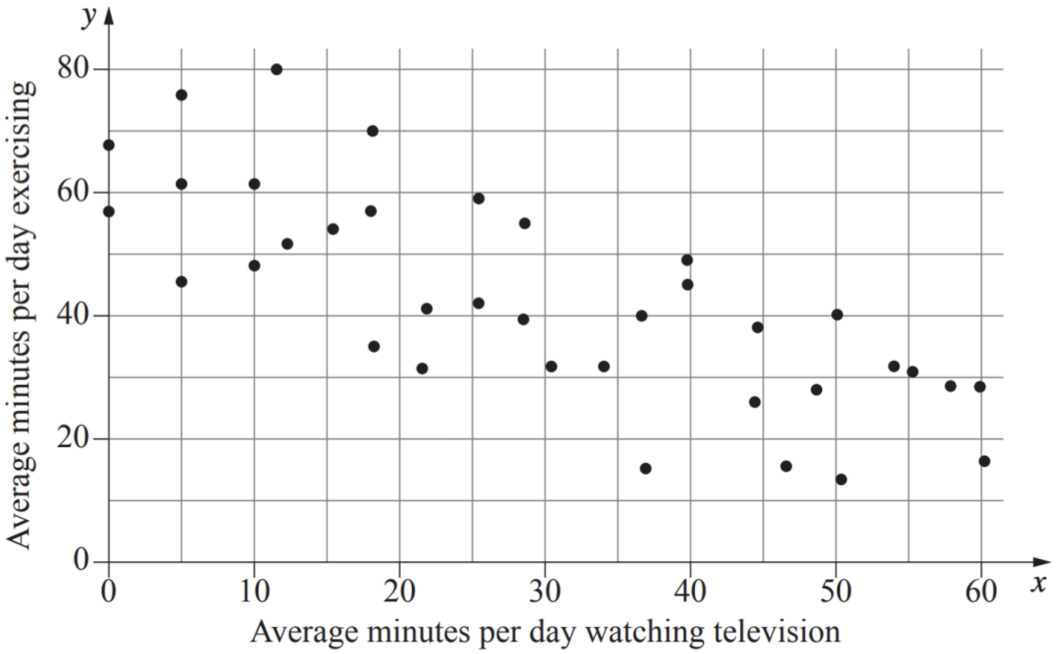
$r = \sqrt[3]{27} = 3$

$p = T_2 = ar = 75 \times 3 = 225$

$q = T_3 = ar^2 = 75 \times 3^2 = 675$ ✓

Question 14 (6 marks)

In a research study, participants were asked to record the number of minutes they spent watching television and the number of minutes they spent exercising each day over a period of 3 months. The averages for each participant were recorded and graphed.



(a) Describe the bivariate dataset in terms of its form and direction. **2**

Form: Linear

Direction: Negative

The equation of the least-squares regression line for this dataset is

$$y = \underset{\substack{\uparrow \\ \text{gradient}}}{m}x + \underset{\substack{\uparrow \\ \text{y-intercept}}}{c}$$

$$y = 64.3 - 0.7x.$$

$$y = -0.7x + 64.3$$

slope = m = −0.7

y-int = c = 64.3

(b) Interpret the values of the slope and y-intercept of the regression line in the context of this dataset. **2**

slope: For every extra minute spent watching TV per day, participants spend 7 minutes less exercising. (As x goes up by 1, y goes down by 7.)

y-intercept: If someone doesn't watch any TV, then they are expected to exercise for 64.3 minutes per day. ($y = 64.3$ for $x = 0$)

(c) Jo spends an average of 42 minutes per day watching television. 1

↑ x

Use the equation of the regression line to determine how many minutes on average Jo is expected to exercise each day.

From part(b) → $y = 64.3 - 0.7x$

$= 64.3 - 0.7(42)$

$= 34.9$ minutes

* You can check your answer on the scatter graph.

$x = 42$ corresponds with $y = 34.9$

(d) Explain why it is NOT appropriate to extrapolate the regression line to predict the average number of minutes of exercise per day for someone who watches an average of 2 hours of television per day. 1

2 hours = 120 minutes. This is way outside the data range.

Extrapolating is not reliable.

According to the model, for $x = 120$

$y = 64.3 - 0.7(120) = -19.7$ minutes per day, which is impossible!!

Alternative answer.

End of Question 14

Question 15 (6 marks)

A sound wave can be modelled using a function $P(t) = k\sin at$, where P is air pressure in Pascals, t is time in milliseconds (ms) and k and a are constants.

(a) Write the equation for a sound wave $P_1(t)$ that has an amplitude of 2 Pascals and a period of 5 ms. **2**

Amplitude $= 2$, $\therefore k = 2$

Period $= \frac{2\pi}{a} = 5$ $\therefore a = \frac{2\pi}{5}$

Equation: $P(t) = 2\sin\left(\frac{2\pi t}{5}\right)$

CALCULATOR.

CHECK

radians set in catalog tool.

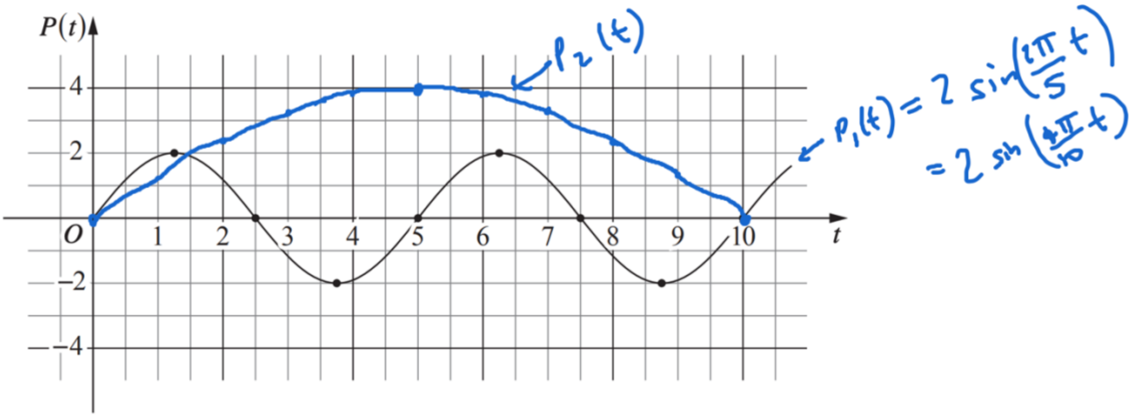
x	$g(x) = 4\sin\left(\frac{\pi^r}{10}x\right)$
0	0
1	1.23..
2	2.35..
3	3.23..
4	3.80..
5	4
6	3.80..
7	3.73..
8	2.35..
9	1.23..
10	0

Amplitude of $P_1(t) = 4$, double the amplitude of $P_1(t)$

Period of $P_2(t) = \frac{2\pi}{\left(\frac{\pi}{10}\right)} = 20$ [4 times the period of $P_1(t)$]

(b) The graph of $P_1(t)$ from part (a) is shown. **2**

On the diagram, sketch the graph of $P_2(t) = 4\sin\left(\frac{\pi}{10}t\right)$ for $0 \le t \le 10$.



Can put $P_2(t)$ as $g(x) = 4\sin\left(\frac{\pi}{10}x\right)$ in TABLE MODE:

Table Range: Start: 0
End: 10
Step: 1

Make sure calculator is set on radians.

(c) Hence, find the values of t, where $0 < t < 10$, for which functions $P_1(t)$ and $P_2(t)$ are BOTH decreasing. **2**

Inspect the graphs to see when both curves have a negative gradient. Turning points for $P_1(t)$ are at 6.25 and 8.75.

$6.25 < t < 8.75$

End of Question 15

Question 16 (5 marks)

$u = x^2 \quad \frac{du}{dx} = 2x$

$v = e^x \quad \frac{dv}{dx} = e^x$

Consider the function $f(x) = \frac{x^2}{e^x}$. $= \frac{u}{v}$

(a) Find the stationary points of the function and determine their nature. **4**

$$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{e^x \cdot 2x - x^2 \cdot e^x}{(e^x)^2}$$

$$= \frac{x\cancel{e^x}(2-x)}{\cancel{e^x}\cdot e^x}$$

$$= \frac{x(2-x)}{e^x}$$

Stationary points occur when:

$f'(x) = 0$

$\frac{x(2-x)}{e^x} = 0$

$\Rightarrow x = 0$ or $x = 2$

$f(0) = \frac{0}{e^0} = 0 \qquad f(2) = \frac{2^2}{e^2} = \frac{4}{e^2}$

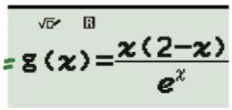
$\therefore$ stat points are: $\underset{\uparrow \atop \text{min}}{(0,0)}$ and $\underset{\uparrow \atop \text{max}}{\left(2, \frac{4}{e^2}\right)}$

TABLE mode provides a table of values for f(x) and f'(x) in the domain: $-1 \le x \le 7$

```
CASIO
f(x)=x²/eˣ
```

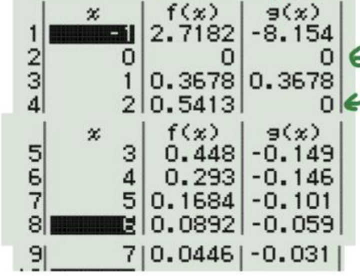
```
Table Range
 Start:-1
 End  :7
 Step :1
```

$f'(x) = g(x) = \frac{x(2-x)}{e^x}$



```
g(x)=x(2−x)/eˣ
```

f'(x):



```
    x     f(x)    g(x)
1  -1  2.7182  -8.154
2   0       0       0
3   1  0.3678  0.3678
4   2  0.5413       0
    x     f(x)    g(x)
5   3   0.448  -0.149
6   4   0.293  -0.146
7   5  0.1684  -0.101
8   6  0.0892  -0.059
9   7  0.0446  -0.031
```

← minimum stationary point

← maximum stationary point

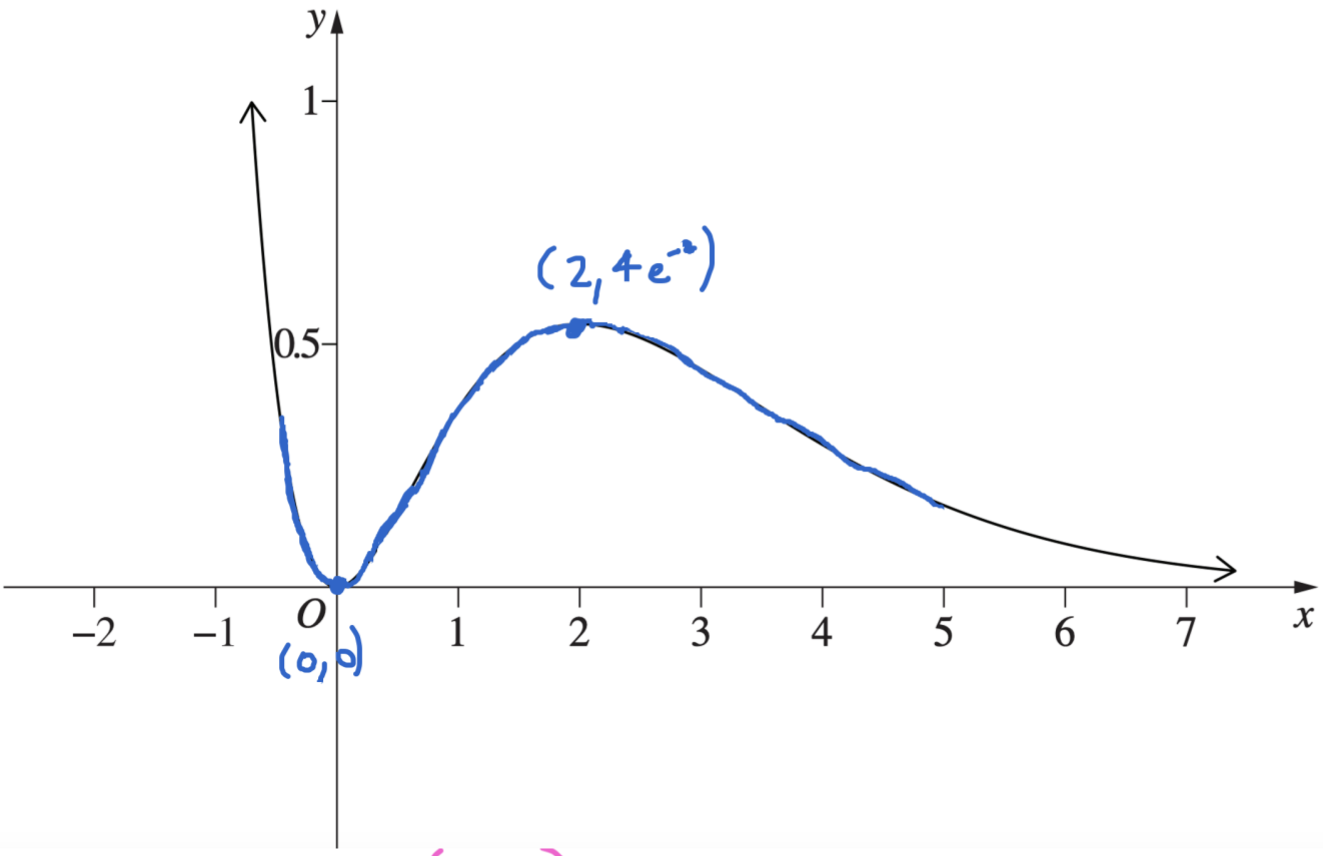
Note: both f(x) and f'(x) values show min and max turning points at (0,0) and (2, 0.5413)

Table for graph of f(x) in Q16b below:

x	-1	0	1	2	3	4	5	6	7
$f(x)$	2.7	0	$\frac{1}{e} \simeq 0.4$	$\frac{4}{e^2} \simeq 0.54$	0.45	0.29	0.17	0.09	0.04
$f'(x)$	-8.2	0	0.37	0	-0.15	-0.15	-0.10	-0.06	-0.03

(b) A partially completed graph of $f(x) = \dfrac{x^2}{e^x}$ is shown. **1**

Use your answer from part (a) to complete the graph.



Local min at $(0,0)$.

Local max at $(2, 4e^{-2}) \simeq (2, 0.54)$

Question 17 (7 marks)

A borrower obtains a reducing-balance loan of \$800 000 to buy a house.

Interest is charged at 0.5% monthly, compounded monthly.

On the last day of each month, interest is added to the balance owing on the loan and then the monthly repayment of \$5740 is made.

Let $\$A_n$ be the balance owing on the loan at the end of n months.

(a) Show that $A_2 = 800\,000(1.005)^2 - 5740(1.005) - 5740$. **2**

Interest is charged at 0.5% monthly, compounded monthly.

$r = 0.5\% = \frac{0.5}{100} = 0.005$. Initial value = \$800 000

$\therefore A_0 = 800\,000$ Repayment = \$5740

$A_1 = A_0 \times 1.005 - 5740$

$= 800\,000(1.005) - 5740$ ✓

$A_2 = A_1 \times 1.005 - 5740$

$= [800000 \times 1.005 - 5740] \times 1.005 - 5740$

$= 800\,000(1.005)^2 - 5740(1.005) - 5740$. ✓

(b) Show that $A_n = 1\,148\,000 - 348\,000(1.005)^n$. From part (a): **3**

$A_3 = A_2 \times 1.005 - 5740$

$= [800\,000(1.005)^2 - 5740(1.005) - 5740] \times 1.005 - 5740$

$= 800000(1.005)^3 - 5740[1 + 1.005 + 1.005^2]$

Continuing the pattern:

$A_n = 800000(1.005)^n - 5740[1 + 1.005 + 1.005^2 + \dots + 1.005^{n-1}]$

$= 800000(1.005)^n - 5740 \times \left[\frac{1.005^n - 1}{1.005 - 1}\right]$

$= 800000(1.005)^n - \frac{5740}{0.005}(1.005^n - 1)$

$= 800000(1.005)^n - 1\,148\,000(1.005^n - 1)$

$= (800000 - 1148000)1.005^n + 1148000$

$= -348\,000(1.005)^n + 1148000$

$= 1148000 - 348000(1.005)^n$

(c) After how many months will the balance owing on the loan first be less than $400 000? **2**

Need $A_n < 400\,000$

$$1148\,000 - 348\,000(1.005)^n < 400\,000$$

$$-348\,000(1.005)^n < -748\,000$$

$$1.005^n > \frac{-748\,000}{-348\,000}$$

$$n \ln 1.005 > \ln\left(\frac{748}{348}\right)$$ ✓

$$n > \frac{0.7652004\ldots}{\ln 1.005}$$

$$n > 153.42\ldots$$

$\therefore n = 154$ months ✓

(12 years and 10 months)

End of Question 17

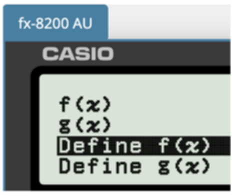
Can use calculator TABLE mode to check answer, or if you struggle with logs.

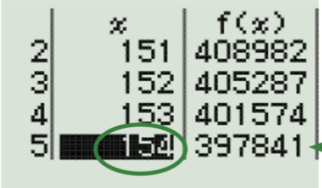
Very quickly you can see that $150 < n < 160$.

Set table range: $\boxed{000}$
- start : 150
- End : 160
- Step : 1

Define $f(x) = 1148\,000 - 348\,000 \times 1.005^x$

Search for first value of $f(x) < 400\,000$


fx-8200 AU
CASIO
f(x)
g(x)
Define f(x)
Define g(x)



	x	f(x)
2	151	408982
3	152	405287
4	153	401574
5	154	397841

$f(x) < 400\,000$

for $n = x = 154$

domain = possible x-values.
range = possible y-values.

This is the hyperbola $f(x)=\frac{3}{x-1}$ translated vertically by 5 units.

Question 18 (2 marks)

Find the range of $g(f(x))$, given $f(x)=\frac{3}{x-1}$ and $g(x)=x+5$. **2**

$y = g\left[f(x)\right] = \left(\frac{3}{x-1}\right) + 5$ note: $x \neq 1$ $\frac{3}{x-1} \neq 0$

As $x \to \infty$ $y \to 5^+$
As $x \to -\infty$ $y \to 5^-$
So $y \neq 5$

∴ range is All real y, $y \neq 5$
OR $(-\infty, 5) \cup (5, \infty)$

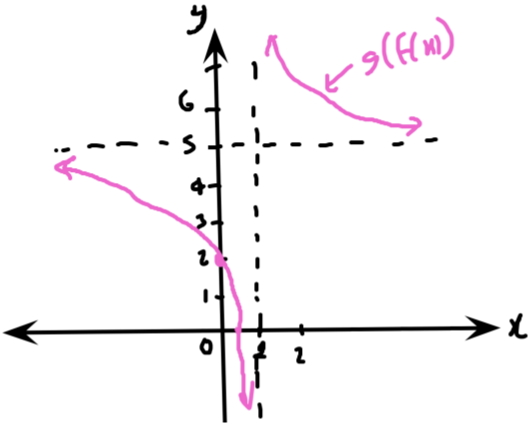
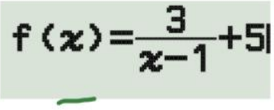
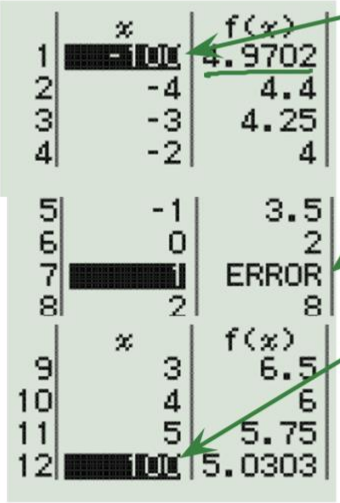


TABLE mode can help check range for $y = \frac{3}{x-1} + 5$



$f(x)=\frac{3}{x-1}+5$



```
      x      f(x)
 1  -100   4.9702
 2    -4      4.4
 3    -3     4.25
 4    -2        4
 5    -1      3.5
 6     0        2
 7     1    ERROR
 8     2        8
      x      f(x)
 9     3      6.5
10     4        6
11     5     5.75
12   100   5.0303
```

For $x \to -\infty$ enter a large negative value. $y \to 5^-$

Error for a y-value may indicate an asymptote at $x = 1$

For $x \to \infty$ enter a large value of x, such as $x = 100$. $y = 5.03..$ confirms $y \to 5^+$

Probability of selection:

$P(A) = \frac{1}{6}$

$P(B) = \frac{2}{6} = \frac{1}{3}$

$P(C) = \frac{3}{6} = \frac{1}{2}$

Question 19 (3 marks)

Three girls, Amara, Bala and Cassie, have nominated themselves for the local soccer team. Exactly one of the girls will be selected. The chances of their selection are in the ratio 1 : 2 : 3, respectively. **3**

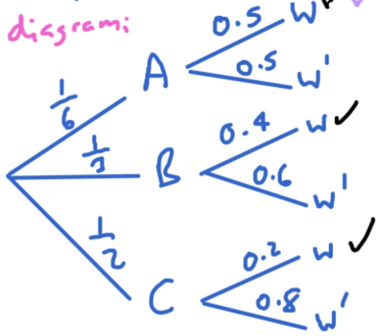
$1 + 2 + 3 = 6$ parts

Let W be the event that the team wins. ✓

The probability that the team wins when:

- Amara is selected is 0.5 $P(W|A) = 0.5$
- Bala is selected is 0.4 $P(W|B) = 0.4$
- Cassie is selected is 0.2. $P(W|C) = 0.2$

Tree diagram:



Given that the team wins, find the probability that Amara was selected.

$$P(A|W) = \frac{P(A \cap W)}{P(W)} ✓ ✓$$

$$= \frac{\frac{1}{6} \times 0.5}{\left(\frac{1}{6} \times 0.5 + \frac{1}{3} \times 0.4 + \frac{1}{2} \times 0.2\right)}$$

$$= \frac{5}{19}$$

Question 20 (3 marks)

The table shows future value interest factors for an annuity of \$1. **3**

Period (*n*) \ Rate (*r*)	*0.005*	*0.01*	*0.015*	*0.02*	*0.03*	*0.06*
7	7.10588	7.21354	7.32300	7.43428	7.66246	8.39384
28	29.97452	32.12910	34.48148	37.05121	42.93092	68.52811
56	64.44140	74.58098	86.79754	101.55826	141.15377	418.82235
84	104.07393	130.67227	166.17264	213.86661	365.88054	2209.41674

Lin invests a lump sum of \$21 000 for 7 years at an interest rate of 6% per annum, compounding monthly.

Yemi wants to achieve the same future value as Lin by using an annuity. Yemi plans to deposit a fixed amount into an investment account at the end of each month for 7 years. The investment account pays 6% per annum, compounding monthly.

Using the table provided, determine how much Yemi needs to deposit each month.

For both Yin and Lemi:

$n = 7$ years

$= 7 \times 12$ months

$= 84$ months

$r = 6\%$ p.a

$= \frac{6}{12}\%$ per month

$= 0.5\%$ per month

$= 0.005$

Lin : $FV = PV(1+r)^n$

$= 21000\,(1+0.005)^{84}$

$= 31927.762\ldots$

$\simeq \$31927.76$

Yemi : $FV = \$31927.76$ FV interest factor $= 104.07393$

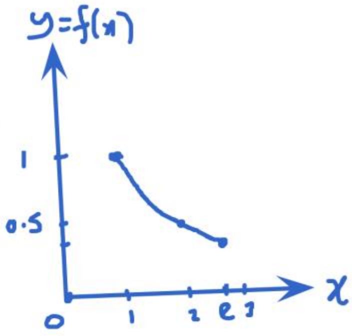
$\therefore$ Monthly deposit $= \dfrac{\$31927.76}{104.07393}$

$= \$306.779\ldots$

$\simeq \$306.78$

Question 21 (5 marks)

A continuous random variable X has a probability density function given by



$$f(x)=\begin{cases} 0 & x<1 \\ \dfrac{1}{x} & 1\le x\le e\,. \\ 0 & x>e \end{cases}$$

$e \simeq 2.72$ $\quad f(1)=1,\ f(e)=\frac{1}{e}\approx 0.37$

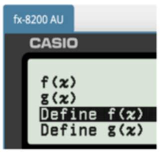
(a) Find the mode of the given probability density function. Justify your answer. **2**

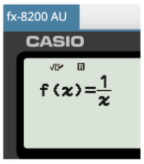
The mode is the x-value of the maximum value of the graph.

Since $f(x)$ is decreasing for $1\le x\le e$, the maximum occurs at $x=1$.

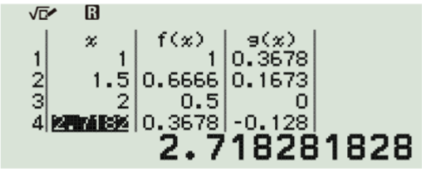
$\therefore$ mode $=1$

You can check using TABLE mode: Press shift 8 to set exact value of e into the table.





$f(x)=\frac{1}{x}$



	x	f(x)	g(x)
1	1	1	0.3678
2	1.5	0.6666	0.1673
3	2	0.5	0
4	2.7182	0.3678	-0.128

2.718281828

(b) Calculate the value of the 25th percentile (Q_1) of this distribution. Give your answer correct to 3 decimal places. **3**

1st quartile $= Q_1 = k$ occurs when:

$$\int_0^k f(x)\,dx = 0.25$$

i.e. $$\int_1^k \frac{1}{x}\,dx = 0.25$$

$$\ln(k) - \ln(1) = 0.25$$

$$\ln k - 0 = 0.25$$

$$k = e^{0.25}$$

$$= 1.28402\ldots$$

$$\simeq 1.28 \quad (3\text{dp})$$

Question 22 (2 marks)

Prove that **2**

$$\frac{\sin^4\theta + \cos^4\theta}{\sin^2\theta\cos^2\theta} + 2 = \sec^2\theta\,\text{cosec}^2\theta.$$

$$LHS = \frac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\sin^4\theta + 2\sin^2\theta\cos^2\theta + \cos^4\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1^2}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta} \times \frac{1}{\cos^2\theta}$$

$$= \text{cosec}^2\theta \times \sec^2\theta$$

$$= \sec^2\theta\cos^2\theta$$

$$= RHS$$

Question 23 (5 marks)

(a) In a flock of 12 600 sheep, the ratio of males to females is 1 : 20. **4**

M : F ← 1 + 20 = 21 parts

$\frac{1}{21}$ $\frac{20}{21}$

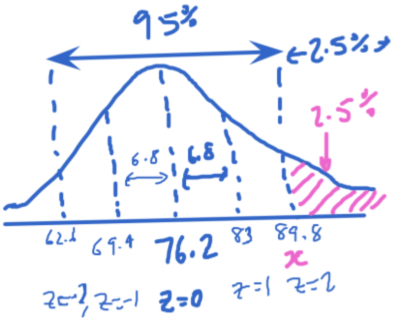
The weights of the male sheep are normally distributed with a mean of 76.2 kg and a standard deviation of 6.8 kg.

↑ σ ↑ μ

In the flock, 15 of the male sheep each weigh more than x kg.

Find the value of x.

Male sheep:



Number of male sheep $= \frac{1}{21} \times 12600 = 600$

Percentage of males weighing more than x kg $= \frac{15}{600} \times 100\%$

$= 2.5\%$

According to the reference sheet 95% of scores have z-scores between -2 and 2. So $\frac{5\%}{2} = 2.5\%$ of scores are above $z = 2$.

So by the rule $z = \frac{x - \mu}{\sigma} \Rightarrow x = \mu + z\sigma$

$x = 76.2 + 2 \times 6.8$

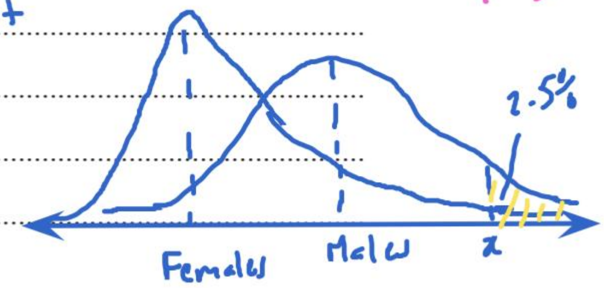
$= 89.8$ kg

(b) The weights of the female sheep are also normally distributed but have a smaller mean and smaller standard deviation than the weights of male sheep. **1**

Explain whether it could be expected that 300 of the females from the flock each weigh more than x kg, where x is the value found in part (a).

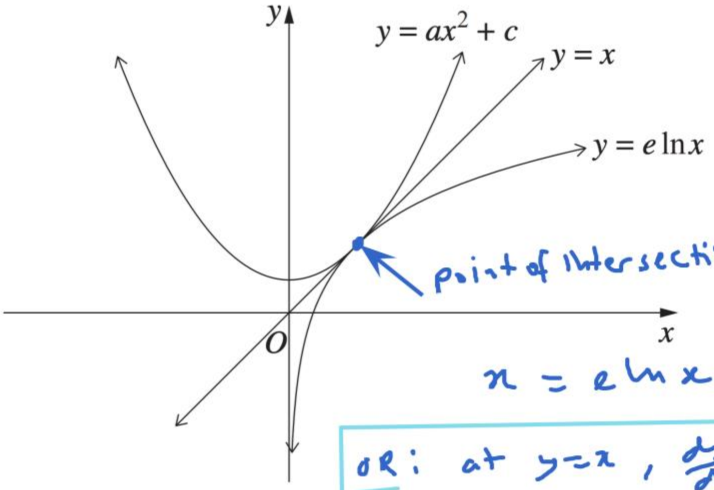
300 female sheep from the flock corresponds to $\frac{300}{12000} = 2.5\%$, which corresponds to a z-score of 2. Since the mean and standard deviation are smaller for females, the z-score for the x-value will be $\left(z_F = \frac{x - \mu_F}{\sigma_F}\right)$ larger. So less than 2.5% of females will weigh more than x.

∴ It is not expected that 300 females weigh more than x.



Question 24 (4 marks)

The graphs of $y = e \ln x$ and $y = ax^2 + c$ are shown. The line $y = x$ is a tangent to both graphs at their point of intersection. **4**



point of intersection :

$$x = e \ln x \Rightarrow x = e$$

OR: at $y = x$, $\frac{dy}{dx} = 1$

$$\frac{d}{dx}(e \ln x) = e \times \frac{1}{x} = 1$$

$$\frac{e}{x} = 1 \Rightarrow x = e$$

Find the values of a and c.

Gradients are the same:

$$\therefore \frac{d}{dx}(ax^2 + c) = 2ax$$

At $x = e$, gradient $= 1$

$$\therefore 2ae = 1$$

$$a = \frac{1}{2e}$$

To find c, substitute $x = e$ and $a = \frac{1}{2e}$ at intersection of the parabola $(y = ax^2 + c)$ and line $(y = x)$:

$$ax^2 + c = x$$

$$\left(\frac{1}{2e}\right) \cdot (e)^2 + c = e$$

$$\frac{e}{2} + c = e$$

$$c = e - \frac{e}{2}$$

$$c = \frac{e}{2}$$

Question 25 (6 marks)

(a) Show that

$$\frac{d}{dx}(\sin x - x\cos x) = x\sin x.$$

2

Product rule:

$y = u \times v$

$u = x \quad u' = 1$

$v = \cos x \quad v' = -\sin x$

$y' = uv' + vu'$

$$\frac{d}{dx}(\sin x - x\cos x) = \cos x - [x(-\sin x) + \cos x\,(1)]$$

$$= \cos x + x\sin x - \cos x$$

$$= x\sin x$$

(b) Hence, find the value of $\int_0^{2025\pi} x\sin x\,dx$. 2

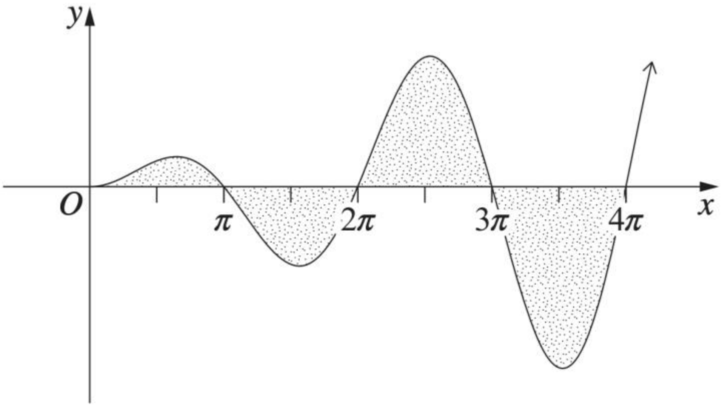
$$\int_0^{2025\pi} x\sin x\,dx = \left[\sin x - x\cos x\right]_0^{2025\pi}$$ from part (a)

$$= (\sin 2025\pi - 2025\pi\cos 2025\pi) - (\sin 0 - 0\cos 0)$$

$$= 0 - 2025\pi(-1) - 0$$

$$= 2025\pi$$

(c) The regions bounded by the x-axis and the graph of $y = x\sin x$ for $x \geq 0$ are shown. **2**



Let $A_n = \int_{(n-1)\pi}^{n\pi} x\sin x \, dx$, where n is a positive integer.

It can be shown that $|A_n| = (2n-1)\pi$. (Do NOT prove this.)

Find the exact total area of the regions bounded by the curve $y = x\sin x$, and the x-axis between $x = 0$ and $x = 2025\pi$.

Total area $= |A_1| + |A_2| + |A_3| + \ldots + |A_{2025}|$

$= \pi + 3\pi + 5\pi + \ldots\ldots + 4049\pi$

$= \pi(1+3+5+\ldots+4049)$

$= \pi \times S_{2025}$

$= \pi \times \frac{2025}{2}(1+4049)$

$= \pi \times \frac{2025}{2} \times 4050$

$= (2025)^2\pi$

$= 4100625\pi$ units^2

NOTE:

$|A_{2025}| = (2\times2025-1)\pi$

$= 4049\pi$

Arithmetic series:

$a=1$, $d=2$, $n=2025$

$l=4049$

$S_n = \frac{n}{2}(a+l)$

Question 26 (5 marks)

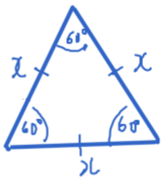
A piece of wire is 100 cm long. Some of the wire is to be used to make a circle of radius r cm. The remainder of the wire is used to make an equilateral triangle of side length x cm.

(a) Show that the combined area of the circle and equilateral triangle is given by **2**

$$A(x) = \frac{1}{4}\left(\sqrt{3}x^2 + \frac{(100-3x)^2}{\pi}\right).$$

Area =

+



Perimeters add to 100cm

$2\pi r + 3x = 100$

$2\pi r = 100 - 3x$

$r = \dfrac{100-3x}{2\pi}$

$= \pi r^2 + \frac{1}{2} ab \sin C$

$= \pi \left(\dfrac{100-3x}{2\pi}\right)^2 + \frac{1}{2} \times x \times x \times \sin 60°$

$= \dfrac{\pi \times (100-3x)^2}{4\pi^2} + \frac{1}{2} x^2 \times \frac{\sqrt{3}}{2}$

$= \dfrac{(100-3x)^2}{4\pi} + \dfrac{\sqrt{3}\,x^2}{4}$

$= \dfrac{1}{4}\left[\sqrt{3}\,x^2 + \dfrac{(100-3x)^2}{\pi}\right]$

(b) By considering the quadratic function in part (a), show that the maximum value of $A(x)$ occurs when all the wire is used for the circle. **3**

$$A(x) = \frac{1}{4}\left[\sqrt{3}\,x^2 + \frac{10000 - 600x + 9x^2}{\pi}\right]$$

$$= \frac{1}{4}\left(\sqrt{3}+\frac{9}{\pi}\right)x^2 - \frac{600}{4\pi}x + \frac{10000}{4\pi}$$

$$= \frac{1}{4}\left(\sqrt{3}+\frac{9}{\pi}\right)x^2 - \frac{150}{\pi}x + \frac{2500}{\pi}$$

This parabola is concave up since $a > 0$. $\therefore$ the turning point is a minimum.

So the maximum value is at the endpoints. NOTE: $x \geq 0$. Also $r \geq 0 \Rightarrow 100 - 3x \geq 0$

$100 \geq 3x$

$\frac{100}{3} \geq x$

$\therefore 0 \leq x \leq \frac{100}{3}$

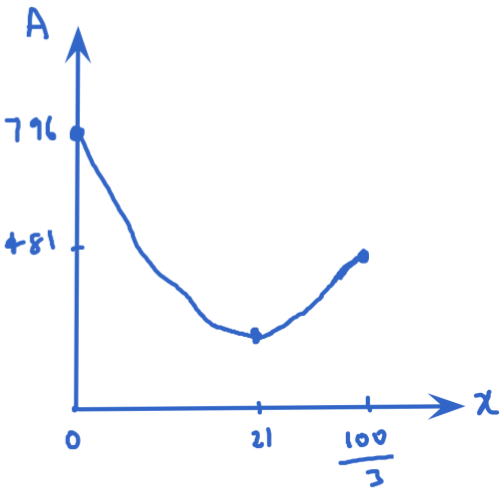
At $x = 0$, Area $= 0 - 0 + \frac{2500}{\pi} \simeq 795.77\ldots$

At $x = \frac{100}{3}$ $(r = 0)$ Area $= \pi r^2 + \frac{\sqrt{3}}{4}x^2$

$= 0 + \frac{\sqrt{3}}{4}\left(\frac{100}{3}\right)^2$

$= 481.125\ldots$

Maximum area occurs when $x = 0$ and $r = \frac{50}{\pi}$.



That is, when all wire is used for the circle.

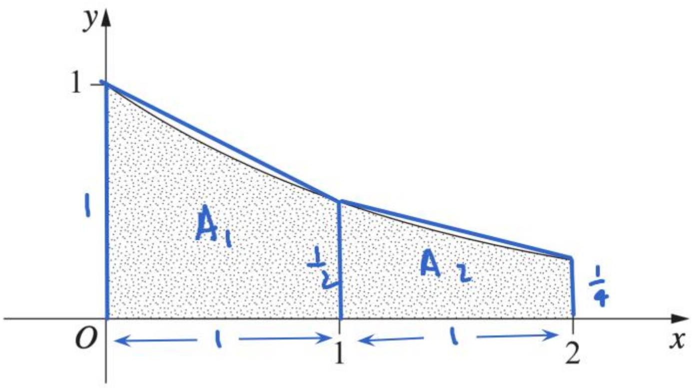
A(x) from part (a) could be placed in TABLE mode for $0 \leq x \leq \frac{100}{3}$.

Question 27 (6 marks)

The shaded region is bounded by the graph $y = \left(\frac{1}{2}\right)^x$, the coordinate axes and $x = 2$.

x	$f(x) = \left(\frac{1}{2}\right)^x$
0	1
1	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

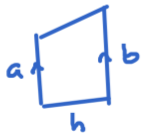
* You can use TABLE mode for this.



(a) Use two applications of the trapezoidal rule to estimate the area of the shaded region. **2**

Area of trapezium

$= \frac{h}{2}(a+b)$



$$\text{Area} = A_1 + A_2$$

$$= \frac{1}{2}\left(1+\frac{1}{2}\right) + \frac{1}{2}\left(\frac{1}{2}+\frac{1}{4}\right)$$

$$= \frac{9}{8}$$

Or by formula: $\text{Area} = \frac{b-a}{2n}\left\{f(a) + f(b) + 2f(x_1)\right\}$

$$= \frac{2-0}{2\times2}\left\{1 + \frac{1}{4} + 2\times\frac{1}{2}\right\}$$

$$= \frac{9}{8} \text{ units}^2$$

(b) Show that the exact area of the shaded region is $\frac{3}{4\ln 2}$. **2**

$$\int a^x\,dx = \frac{a^x}{\ln a} + C$$

$$\text{Exact area} = \int_0^2 \left(\tfrac{1}{2}\right)^x dx$$

$$= \left[\frac{\left(\frac{1}{2}\right)^x}{\ln\left(\frac{1}{2}\right)}\right]_0^2$$

$$= \frac{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^0}{\ln\left(\frac{1}{2}\right)}$$

$$= \frac{\frac{1}{4} - 1}{\ln 2^{-1}}$$

$$= \frac{-\frac{3}{4}}{-\ln 2}$$

$$= \frac{3}{4\ln 2}$$

(c) Using your answers from part (a) and part (b), deduce $e < 2\sqrt{2}$. **2**

Exact area < trapezoidal approximation (because curve is concave up)

$$\frac{3}{4\ln 2} < \frac{9}{8}$$

$$24 < 36\ln 2$$

$$2 < 3\ln 2$$

$$2 < \ln 2^3$$

$$2 < \ln 8$$

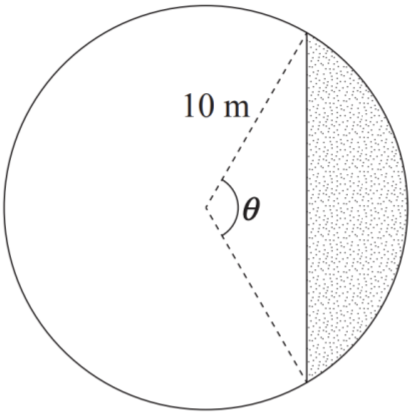
$$e^2 < 8$$

i.e. $e < \sqrt{8}$

$\therefore\ e < 2\sqrt{2}$

Question 28 (4 marks)

A farmer wants to use a straight fence to divide a circular paddock of radius 10 metres into two segments. The smaller segment is $\frac{1}{4}$ of the paddock and is shaded in the diagram. The fence subtends an angle of θ radians at the centre of the circle as shown.



NOT TO SCALE

(a) Show that $\theta = \sin\theta + \frac{\pi}{2}$. **2**

Shaded area = area of sector − area of triangle

$\frac{1}{4} \times$ area of circle $= \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C$

$$\frac{1}{4}\times\pi\times10^2 = \frac{1}{2}\times10^2\times\theta - \frac{1}{2}\times10\times10\times\sin\theta$$

$$25\pi = 50\theta - 50\sin\theta$$

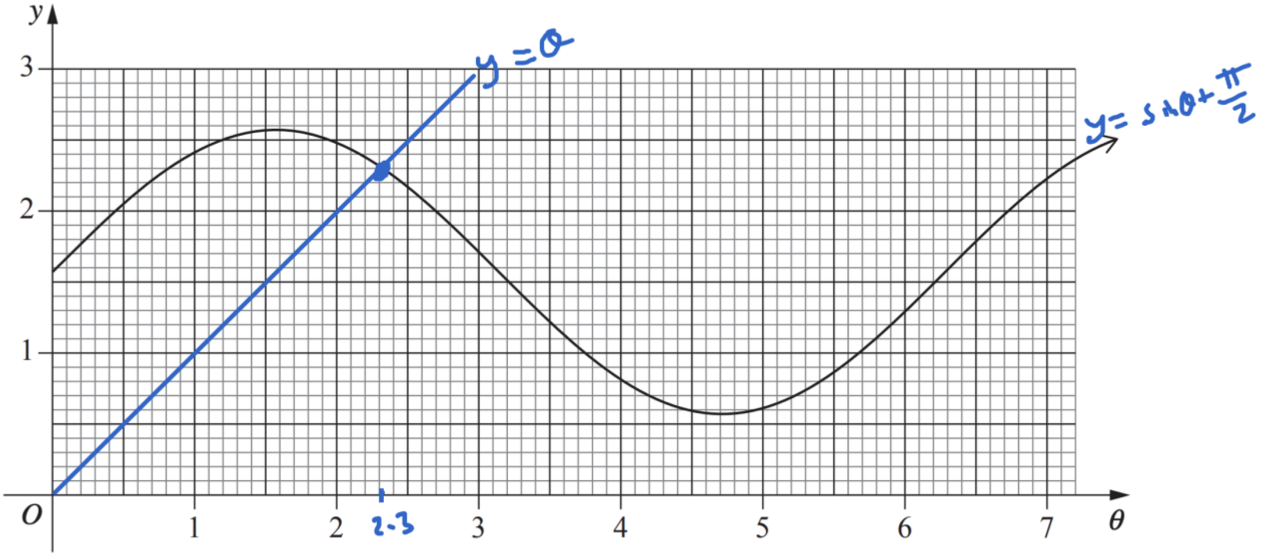
$$25\pi = 50(\theta - \sin\theta)$$

$$\frac{25\pi}{50} = \theta - \sin\theta$$

$$\frac{\pi}{2} = \theta - \sin\theta$$

$$\therefore \theta = \sin\theta + \frac{\pi}{2}$$

(b) The graph of $y = \sin\theta + \frac{\pi}{2}$ is shown. From (a): $\theta = \sin\theta + \frac{\pi}{2}$ **2**



Use the graph and the result in part (a) to estimate the arc length of the smaller segment to the nearest metre.

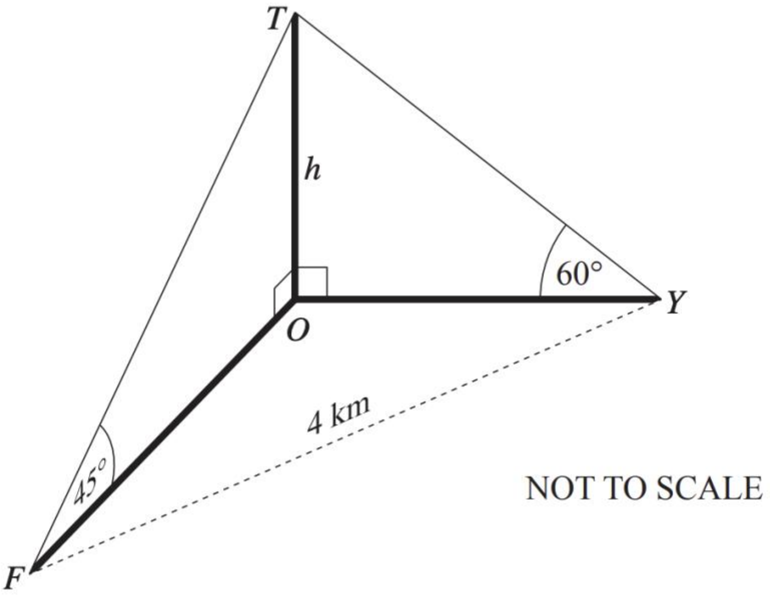
arc length = $l = r\theta$ $r = 10\text{ m}$, $\theta \approx 2.3$ radians from graph

$l = 10 \times 2.3$

$= 23\text{ m}$

Question 29 (7 marks)

The point T is the peak of a mountain and the point O is directly below the mountain's peak. The point Y is due east of O and the angle of elevation of T from Y is 60°. The point F is 4 km south-west of Y. The points O, Y and F are on level ground. The angle of elevation of T from F is 45°.



(a) Let the height of the mountain be h. **1**

Show that $OY = \dfrac{h}{\sqrt{3}}$.

Since $\angle TOY = 90°$:

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 60° = \frac{h}{OY}$$

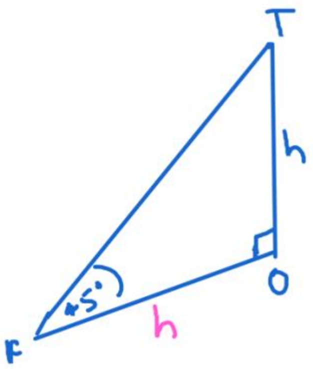
$$\sqrt{3} = \frac{h}{OY}$$

If calculator settings input/output is set to MathI/MathO then calculator will give exact surd answers for trig functions. i.e. $\tan 60° = \sqrt{3}$

```
tan(60)
                √3
```

Question 29 (continued)

(b) Hence, or otherwise, find the value of *h*, correct to 2 decimal places. **3**



In $\triangle TOF$, angle sum of $\triangle = 180°$

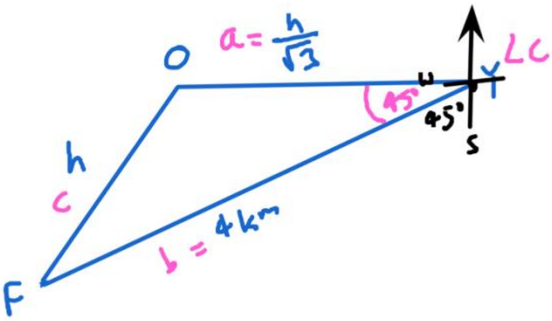
$\therefore \angle FTO + \angle OFT + \angle TOF = 180°$

$\angle FTO + 45° + 90° = 180°$

$\therefore \angle FTO = 45°$

and the triangle is isosceles.

$\therefore FO = OT = h$



The point F is SW of Y.

$\therefore \angle OYF = 45°$

This question now involves 3 sides and an angle, ∴ we cosine rule.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$OF^2 = OY^2 + FY^2 - 2 \times OY \times FY \times \cos 45°$$

$$h^2 = \left(\frac{h}{\sqrt{3}}\right)^2 + 4^2 - 2 \times \left(\frac{h}{\sqrt{3}}\right) \times 4 \times \frac{\sqrt{2}}{2}$$

$$h^2 = \frac{h^2}{3} + 16 - \frac{4h\sqrt{2}}{\sqrt{3}}$$

$$0 = -\frac{2}{3}h^2 - \frac{4\sqrt{2}}{\sqrt{3}}h + 16$$

$\left(\times -\frac{3}{2}\right):$

$$0 = h^2 + 2\sqrt{6}h - 24$$

Using quadratic formula: $a = 1$ $b = 2\sqrt{6}$ $c = -24$

$$h = \frac{-2\sqrt{6} \pm \sqrt{(2\sqrt{6})^2 - 4 \times 1 \times (-24)}}{2(1)}$$

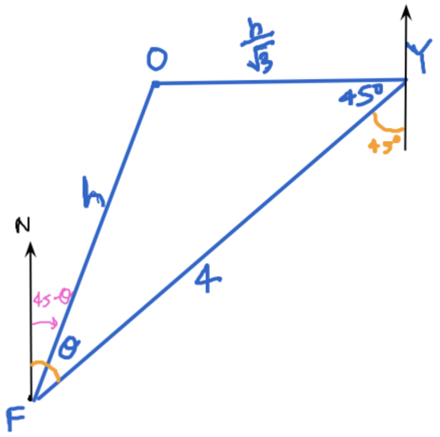
$$= \frac{-2\sqrt{6} \pm \sqrt{120}}{2}$$

$$= -\sqrt{6} \pm \sqrt{30}$$

$h = 3.0277\ldots$ (since $h > 0$)

$\simeq 3.03$ km

(c) Find the bearing of point *O* from point *F*, correct to the nearest degree. **3**



Bearing of O from F is $45° - \theta°$

(First produce alternate angles in parallel lines = 45°)

Use sine rule to find $\theta = \angle$

$$\frac{\sin\theta}{h/\sqrt{3}} = \frac{\sin 45°}{h}$$

$$\sin\theta = \frac{\cancel{h}}{\sqrt{3}} \times \frac{1}{\sqrt{2}\,\cancel{h}}$$

$$= \frac{1}{\sqrt{6}}$$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{6}}\right) = 24.094.. \approx 24°$$

$\therefore$ Bearing $= 45° - 24°$

$= 021°$

Question 30 (3 marks)

The parabola with equation $y = (x - 1)(x - 5)$ is translated both horizontally to the right and vertically up by k units, where k is positive. **3**

The translated parabola passes through the point (6, 11).

Find the value of k.

Horizontal translation: $x \longrightarrow x - k$

Vertical translation: $y \rightarrow y - k$

After both translations the equation becomes:

$$y - k = (x - k - 1)(x - k - 5)$$

The point (6,11) lies on the translated parabola.

Substitute $x = 6$ and $y = 11$ into

$$y = (x - k - 1)(x - k - 5) + k$$

$$11 = (6 - k - 1)(6 - k - 5) + k$$

$$11 = (5 - k)(1 - k) + k$$

$$11 = 5 - 5k - k + k^2 + k$$

$$0 = k^2 - 5k - 6$$

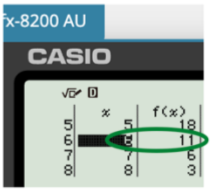
$$(k - 6)(k + 1) = 0$$

$$\Rightarrow k = 6 \text{ or } -1$$

But $k > 0$. So $k = 6$.

Note: You can check on your CASIO calculator by checking in TABLE mode that the graph of the parabola has (6,11) as a solution.

Define $f(x) = (x - 6 - 1)(x - 6 - 5) + 6$



	x	$f(x)$
5	5	18
6	6	11
7	7	6
8	8	3

	x	$f(x)$
8	8	3
9	9	2
10	10	3
11	11	6

Note: Minimum value of the parabola has shifted from (3,−4) to (9,2) has expected.
[6 units to the right and 6 units up]

Question 31 (3 marks)

The equation $\cos px = \frac{1}{2}$ has 2 solutions where $0 \le x \le 2\pi$ and $p > 0$. **3**

$0 \le px \le 2p\pi$

Find all possible values of p.

Let $\theta = px$:

Solutions for $\cos\theta = \frac{1}{2}$

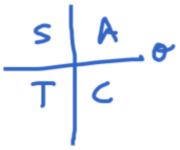
$\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \dots$

$\therefore px = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

So $x = \frac{\pi}{3p}, \frac{5\pi}{3p}, \frac{7\pi}{3p}$

↑ 1st solution ↑ 2nd solution ↖ 3rd solution – not allowed.

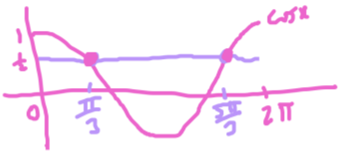
θ is in 1st or 4th quadrant, etc.



① $\frac{\pi}{3}$

④ $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Graph of $\cos x = \frac{1}{2}$ $(p=1)$



2 solutions $\therefore p=1$ is a possible value.

Boundary values for p to get 2 solutions $(0 \le x \le 2\pi)$

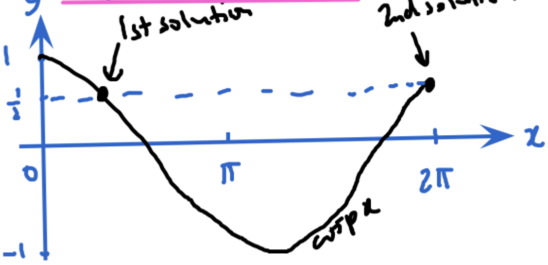
2nd solution is 2π: (Gives boundary for lowest p-value)

$x = \frac{5\pi}{3p} \le 2\pi$

$\Rightarrow 6\pi p \ge 5\pi$

$p \ge \frac{5}{6}$

Graph of $\cos px = \frac{1}{2}$



3rd solution is 2π: (Gives boundary for highest p-value.) (can't have this solution.)

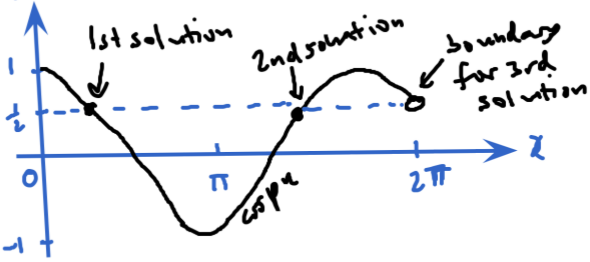
Need $x = \frac{7\pi}{3p} > 2\pi$

$7\pi > 6\pi p$

$p < \frac{7}{6}$

$\therefore \frac{5}{6} \le p < \frac{7}{6}$

Graph of $\cos px = \frac{1}{2}$



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