7 A ten-sided die has faces numbered 1 to 10.

The die is constructed so that the probability of obtaining the number 1 is greater than the probability of obtaining any of the other numbers. The numbers 2 to 10 are equally likely to occur.

When the die is rolled 153 times, a 1 is obtained 72 times.

By using the relative frequency of rolling a 1, which of the following is the best estimate for the probability of rolling a 10?

A. $\frac{1}{17}$

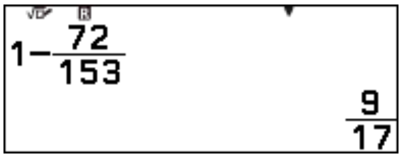
B. $\frac{1}{11}$

C. $\frac{1}{10}$

D. $\frac{1}{9}$

$\text{Prob}(2,\dots,10) = 9p = 1 - \frac{72}{153}$

$\therefore \text{Prob}(\text{"10"}) = p = \frac{1}{17}$



Entering "÷9" calls up the "Ans" (last answer)

8 The minimum daily temperature, in degrees, of a town each year follows a normal distribution with its mean equal to its standard deviation. The minimum daily temperature was recorded over one year.

What percentage of the recorded minimum daily temperatures was above zero degrees?

A. 16%

B. 50%

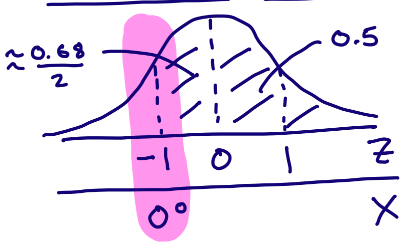
C. 68%

D. 84%

If $x = \sigma$

then $x - \sigma = 0$

So, zero degrees is one sigma below the mean

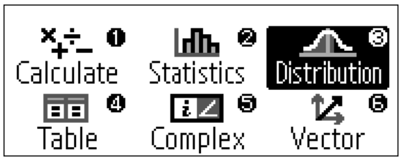


$\Pr(z \geq -1) \approx \frac{0.68}{2} + 0.5$

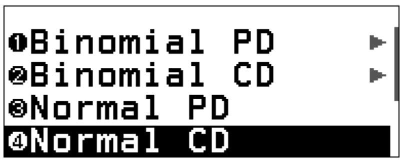
≈ 0.84

or

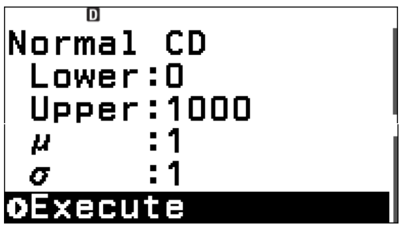
• In the Distribution app



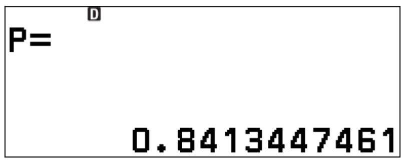
• Press ④ for Normal CD



• Enter • lower = 0
• upper = 1000 (or anything "big")
• $\mu = 1$
• $\sigma = 1$
(or $\mu = 2$, $\sigma = 2$ or similar)



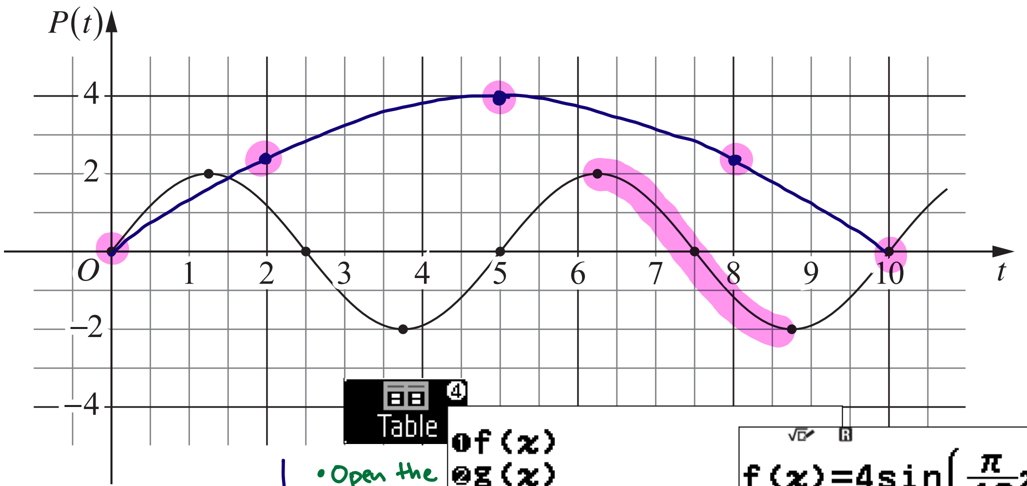
• probability is displayed to 10 decimal places.

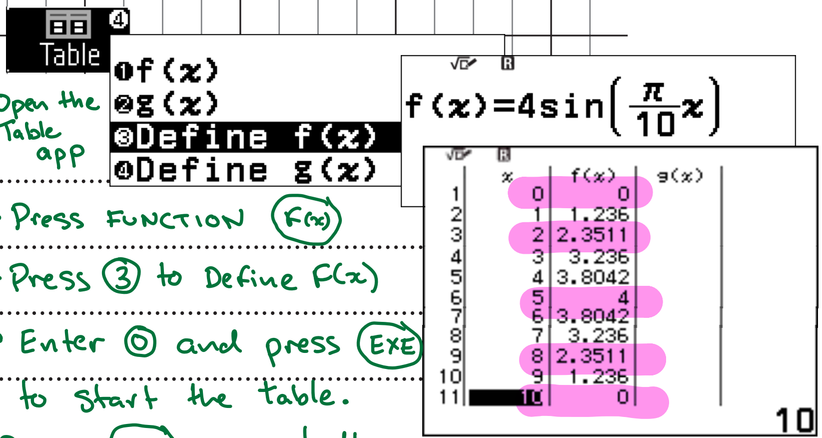


Question 15 (continued)

(b) The graph of $P_1(t)$ from part (a) is shown. **2**

On the diagram, sketch the graph of $P_2(t) = 4\sin\left(\dfrac{\pi}{10}t\right)$ for $0 \le t \le 10$.





$\frac{\pi}{10} = \frac{1}{4} \times \frac{2\pi}{5}$

$\therefore$ period of $P_2(t)$ is $4 \times$ period of $P_1(t)$

($\therefore$ "half" a sine wave shown instead of 2.)

Amplitude of $P_2(t)$ is 4 $\therefore$ max at $y=4$

or

- Open the Table app
- Press FUNCTION (F(x))
- Press ③ to Define F(x)
- Enter ⓪ and press (EXE) to start the table.
- Press (EXE) repeatedly for consecutive x-values
- Plot points incl. max at (5,4) and join with smooth curve

(c) Hence, find the values of t, where $0 < t < 10$, for which functions $P_1(t)$ and $P_2(t)$ are BOTH decreasing. **2**

$6.25 < t < 8.75$

$5 + \frac{5}{4}$ $\qquad$ $5 + \frac{5}{4} \times 3$

based on quarters of 5 the period of $P_1(t)$

End of Question 15

Question 17 (continued)

(b) Show that $A_n = 1\,148\,000 - 348\,000(1.005)^n$. **3**

$A_3 = A_2 \times 1.005 - 5740$

$= 800\,000(1.005)^3 - 5470 \times (1.005)^2 - 5470 \times 1.005 - 5740$

by repeatedly multiplying by 1.005

$A_n = 800\,000 \times 1.005^n - 5470(1.005^{n-1} + 1.005^{n-2} + \dots + 1.005 + 1)$

$= 800\,000 \times 1.005^n - 5470 \times \left(\dfrac{1.005^n - 1}{1.005 - 1}\right)$ Sum of geometric series

$= 800\,000 \times 1.005^n - 1\,148\,000(1.005^n - 1)$

$= 800\,000 \times 1.005^n - 1\,148\,000 \times 1.005^n + 1\,148\,000$

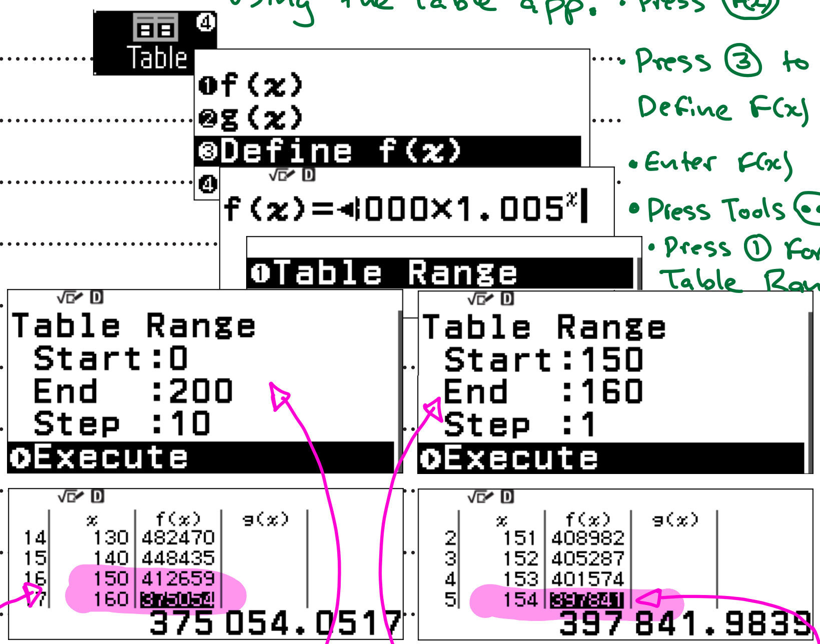
$= 1148000 - 348\,000 \times 1.005^n$ (as required)

(c) After how many months will the balance owing on the loan first be less than \$400 000? **2**

n (months)	A(n)
150	\$412 659
153	\$401 574
154	\$397 842

∴ balance first below \$400 000 after 154 months

Using the Table app: • Press (F(x)) • Press ③ to Define F(x) • Enter F(x) • Press Tools (...) • Press ① for Table Range



• Enter initial range to • locate approx. solution

• Refine range via (...) ① • Find first month with $A_n < 400k$

OR

$1148000 - 348000 \times 1.005^n < 400\,000$

$\therefore 1.005^n > \dfrac{400 - 1148}{-348}$

$\therefore n \times \ln(1.005) > \ln\left(\dfrac{748}{348}\right)$

$n > \ln\left(\dfrac{748}{348}\right) \div \ln 1.005$

$n > 153.42$

∴ balance less than \$400 000 after 154 months

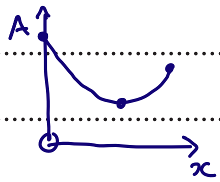
End of Question 17

Question 26 (continued)

(b) By considering the quadratic function in part (a), show that the maximum value of $A(x)$ occurs when all the wire is used for the circle. **3**

$A(x)$ is a quadratic with a positive co-eff of x^2

$\frac{1}{4}\left(\sqrt{3}+\frac{9}{\pi}\right)x^2 - \dots$



$\therefore$ max is at endpoint of domain

Domain is $x \geq 0$ and $r \geq 0 \Rightarrow x \leq \frac{100}{3}$

$A(0) = 795.77\ cm^2$

$A\left(\frac{100}{3}\right) = 481.12\ cm^2$

$\therefore$ max at $x = 0$, which is when the triangle has zero side length and so all the wire is used for the circle.

Using the Table app

4 Table

- Press f(x)
- Press 3

1 f(x)
2 g(x)
3 Define f(x)
4 Define g(x)

$f(x)=\frac{1}{4}\left(\sqrt{3}\,x^2+\frac{(100}{\pi}\right.$

- Enter the function

Table Range
Start :0
End :33.333
Step :3.3333
Execute

- Press Tools
- Press 1 for Table Range

	x	f(x)	g(x)
1	0	795.77	
2	3.3333	649.39	
3	6.6666	528.54	
4	9.9999	433.23	

795.7747155

	x	f(x)	g(x)
8	23.333	307.36	
9	26.666	339.74	
10	29.999	397.66	
11	33.333	481.11	

481.1156019

If $x = \frac{100}{3}$ is not in your table it can be manually entered as an x-value

End of Question 26